## 9 <br> DATA HANDLING

## THEORY

### 9.1 INTRODUCTION

$>$ Primary Data : The data which is collected by the investigator with a definite plan or design in mind is called Primary Data.
$>\quad$ Secondary Data : When the data is gathered from some source(s) which already had stored for some purpose, the data obtained is called Secondary data.
$>\quad$ Ungrouped or Raw Data : The data which is collected for specific purpose and put as it is (without any arrangement) is called raw data. Each entry in raw data is known as an observation.
For example : Runs scored by batsmen in a T-20 cricket match are :

$$
17,20,15,42,25,17,23,18,5,15
$$

By observation we can find the lowest and highest scores.
> Range : The difference between the highest and lowest values of given data is called Range. In the above case
Highest Score $=42$, Lowest Score $=5$
$\therefore \quad$ Range $=$ Highest Score - Score
Range $=42-5=37$
$>\quad$ Frequency : The number of times a particular observation occurs is called its frequency. In the above example, frequency of 15 and 17 is two and for other scores the frequency is 1 .

### 9.2 UNGROUPED AND GROUPED DATA

In our earlier classes, we have already discussed about the ungrouped data. We are also familiar with the terms like raw data, frequency of a variable, frequency distribution, tally method etc.
Now, by taking an example of raw data, we shall deal with representing the ungrouped data into grouped data.
Suppose the marks secured by 40 students of class VIII, out of 100, in the alphabetical order of the names of the students are given below :

| 58, | 34, | 46, | 50, | 34, | 50, | 7, | 5, | 5, | 7, | 18, | 58, | 48, | 77, | 85, | 58, | 34, |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 10, | 5, | 33, | 34, | 45, | 55, | 65, | 48, | 55, | 82, | 50, | 65, | 10, | 18, | 58, | 54, | 45, |
| 30, | 35, | 80, | 58, | 52, | 60 |  |  |  |  |  |  |  |  |  |  |  |

The above data in its original form is called raw data.
To make the tabulation easier and to minimise the number of errors, we can use the tally method.
Tally marks are recorded in bunches of five and the fifth tally mark is drawn diagonally across the first four to make a group of five e.g. $\mathbb{W}$.


| Marks | Tally marks | No. of Students |
| :---: | :---: | :---: |
| 50 | $\|\|\mid$ | 3 |
| 52 | $\mid$ | 1 |
| 54 | $\mid$ | 1 |
| 55 | $\\|$ | 2 |
| 58 | $\mathbb{X}$ | 5 |
| 60 | $\mid$ | 1 |
| 65 | $\\|$ | 2 |
| 77 | $\mid$ | 1 |
| 80 | $\mid$ | 1 |
| 82 | $\mid$ | 1 |
| 85 | $\mid$ | 1 |

Finally, after counting the tally marks against each observation, we write the number of students in the third column against the corresponding variate.
This way of arranging the data is known as ungrouped frequency distribution. Marks are called variates and the number of students who have secured a particular number of marks is called frequency of the variate.
The above ungrouped frequency distribution table can be re-written as follows:

| Class Intervals | Frequency |
| :---: | :---: |
| $1-10$ | 7 |
| $11-20$ | 2 |
| $21-30$ | 1 |
| $31-40$ | 6 |
| $41-50$ | 8 |
| $51-60$ | 10 |
| $61-70$ | 2 |
| $71-80$ | 2 |
| $81-90$ | 2 |
| Total | 40 |

This way of representing frequencies is called the grouped frequency distribution. In this table marks $1-10,11-20$ etc. are called classes or class intervals. These class intervals are called discontinuous class intervals.
1 is called the lower limit and 10 is called the upper limit of the class $1-10$. Class $1-10$ consists of the students securing marks between 1 and 10 (both inclusive) and this group has 7 students class size $=10$ since both marks are included.
Mid point of a class is called its class mark. The class mark of a class is obtained by finding the mean of lower and upper limit of the class interval. For example, the interval 31-40 has class mark

$$
\frac{31+40}{2}=\frac{71}{2}=35.5
$$

Suppose a child has secured 30.5 or 60.5 marks, then in which class interval would the marks lie. In such a situation, we take the class intervals as $0-10,10-20,20-30,30-40$ etc. (lower limit included and upper limit excluded). These class intervals are called continuous class intervals. Thus Marks 30 lie in the interval 30-40 and not in 20-30. Marks 50 lie in the intervals $50-60$ and not in $40-50$. We can re-write the above table having continuous class intervals as follows:

|  | Class Intervals | Frequency |
| :---: | :---: | :---: |
|  | $0-10$ | 5 |
| Marks ' 30 ' not included here | $10-20$ | 4 |
| Marks '30' is included here | $20-30$ | 0 |
|  | $30-40$ | 7 |
| $40-50$ | 5 |  |
| $50-60$ | 12 |  |
| $60-70$ | 3 |  |
| $70-80$ | 1 |  |
|  | $80-90$ | 3 |
|  | Total | 40 |

The class size is 10 i.e. upper class limit - lower class limit.

## Steps to be taken to Prepare a Frequency Table

Step 1 : Arrange the data in array by writing it in ascending or descending order.
Step 2 : Draw a table having three columns: (i) Class interval, (ii) Tally Marks (iii) Frequency.
Step 3 : Considering the lowest and the highest scores, write down the class intervals or variables in column (i) mentioned in step2.
Step 4 : Corresponding to each class interval or variable draw as many strokes in the column (i) as the number of scores it contains.
Step 5 : Count the number of strokes against each interval and write down in column (iii). As already explained, to make the counting easier, we cross the fifth tally mark diagonally.
Thus, we get frequency table.


## Illustration 1

The sale of car batteries of a company per day in a particular month is given below :

| $\mathbf{6 6}$ | $\mathbf{7 6}$ | $\mathbf{8 5}$ | $\mathbf{4 5}$ | $\mathbf{8 7}$ | $\mathbf{9 8}$ | $\mathbf{6 5}$ | $\mathbf{8 2}$ | $\mathbf{8 5}$ | $\mathbf{7 4}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{8 2}$ | $\mathbf{4 7}$ | $\mathbf{9 6}$ | $\mathbf{1 0 0}$ | $\mathbf{5 0}$ | $\mathbf{1 1 0}$ | $\mathbf{8 1}$ | $\mathbf{7 8}$ | $\mathbf{5 2}$ | $\mathbf{4 7}$ |
| $\mathbf{4 8}$ | $\mathbf{7 2}$ | $\mathbf{6 5}$ | $\mathbf{5 8}$ | $\mathbf{8 5}$ | $\mathbf{9 5}$ | $\mathbf{1 0 8}$ | $\mathbf{6 8}$ | $\mathbf{5 8}$ | $\mathbf{7 2}$ |

For the frequency taking one of the class intervals as 80-90

Solution


### 9.3 CUMULATIVE FREQUENCY

The cumulative frequency corresponding to a class is the sum of the all the frequencies upto and including that class.

For example :

| Marks | 0 | 5 | 10 | 14 | 16 | 18 | 20 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of students <br> Frequency | 1 | 4 | 8 | 9 | 12 | 6 | 5 |

From the table we conclude that
1 student secured zero marks
4 students secured 5 marks
8 students secured 10 marks and so on.
Now, how many students secured 10 marks or less.
To answer this question we have to add all the students who secured 10 or less marks
i.e. $(1+4+8)=13$ students.

13 is termed as cumulative frequency of marks 10 . Similarly cumulative frequency of marks 16 is $(1+4+8+9+12)=34$.

### 9.3.1 Type of Cumulative Frequency

There are two types of cumulative frequency table :
(a) Less than type
(b) More than type


## Illustration 2

Construct cumulative frequency distribution (less than type and more than type) from the following data

| Marks obtained | $0-20$ | $20-40$ | $40-60$ | $60-80$ | $80-100$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| No. of students | 2 | 7 | 11 | 18 | 12 |

(a) Less Than Type Cumulative Frequency Distribution Table

| Marks Obtained | No. of Students <br> (Cummulative Frequency) |
| :---: | :---: |
| Less than 20 | 2 |
| Less than 40 | $2+7=9$ |
| Less than 60 | $2+7+11=20$ |
| Less than 80 | $2+7+11+18=38$ |
| Less than 100 | $2+7+11+18+12=50$ |

Here the numbers $20,40,60 \ldots .$. are all upper limits of given class intervals.

## (b) More Than Type Cumulative Frequency Distribution Table

| Marks Obtained | No. of Students <br> (Cummulative Frequency) |
| :---: | :---: |
| More than or equal to 0 | $12+18+11+7+2=50$ |
| More than or equal to 20 | $12+18+11+7=48$ |
| More than or equal to 40 | $12+18+11=41$ |
| More than or equal to 60 | $12+18=30$ |
| More than or equal to 80 | $12=12$ |

Here the numbers $0,20,40,60,80$ are all lower limits of given class intervals.

### 9.4 INCLUSIVE AND EXCLUSIVE GROUPED FREQUENCY DISTRIBUTION

Frequency distribution in which groups are of the form 1-9, 10-19, 20-29.......
In this type of distibtution both the lower limit and upper limit belongs to that class only e.g. for inclusive class $1-9$ both 1 and 9 belongs to class $1-9$.
$10-20,20-30,30-40 \ldots .$. in which upper limit of one class interval is same as the lower class limit of succeeding class interval. This is called exclusive grouped frequency distribution.
An inclusive frequency distribution can be converted to exclusive frequency distribution in the following way:
(i) Find the difference between the upper limit of a class and the lower limit of its succeeding class.
(ii) Add half of this difference to each of the upper limits and substract the same from each of the lower limits.

Class Marks : Class marks are the mid-points of the class intervals.
$\therefore \quad$ Class Marks $=\frac{\text { Lower limit }+ \text { Upper limit }}{2}$
For a class interval $10-20$
Class Mark $=\frac{10+20}{2}=15$

### 9.5 GRAPHICAL REPRESENTATION OF DATA

To makes the data useful and attractive we represent them by means of diagrams and graphs. The representation of quantitative (numerical) data suitably through chart and diagram is known as graphical representation of data.

### 9.5.1 Pictograph

Pictorial representation of numerical data, using picture symbol, is known as pictograph of the data. Most business and industrial concerns show their product by depicting pictures.


## Illustration 3

Read the following numerical data about the total number of students scored more than $\mathbf{9 0 \%}$ aggreagate in annual examination in classes It to V of a school with $\mathbf{1 2 0}$ students in each class having 4 sections and with 30 students in each section.

| Class | I | II | III | IV | V |
| :---: | :---: | :---: | :---: | :---: | :---: |
| No. of students scored <br> more than $90 \%$ aggregate | 10 | 8 | 8 | 7 | 9 |

## Solution

The pictograph of the above data is shown in table, where one student is represented by one $\Delta$.

| I | $\Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta$ | 10 |
| :---: | :---: | :---: |
| II | $\Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta$ | 8 |
| III | $\Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta$ | 8 |
| IV | $\Delta \Delta \Delta \Delta \Delta \Delta \Delta$ | 7 |
| V | $\Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta$ | 9 |

### 9.5.2 Bar Graph

We draw horizontal or vertical bars (rectangles) to represent the given data. Such a representation is called bar graph.
Bar graphs are the simple most popular method of showing data related to variable. Bars are drawn, of uniform width with equal spaces in between them, on X -axis. On Y -axis height of bar (rectangle) is shown.

### 9.5.3 Histograms

A histogram is very similar to bar graph. However, unlike a bargraph there is no gaps between the bars of a histogram. Histograms are used for exclusive (or continuous) for class intervals. In a histogram height of bars are proportional to the frequencies represented.


## Illustration 4

Draw a histogram to represent the following data of the earnings of workers:

| Monthly Earnings <br> (in hundred Rupees) | No.of Workers |
| :---: | :---: |
| $80-120$ | 4 |
| $120-160$ | 6 |
| $160-200$ | 12 |
| $200-240$ | 8 |
| $240-280$ | 5 |

## Solution


(a) On the horizontal axis, mark the class intervals with a uniform scale.
(b) On the vertical axis mark the frequencies, with a uniform scale.
(c) Construct rectangles with class intervals as bases and the corresponding frequencies as heights
In the illustration, the class sizes are same (40), so the rectangles will have the same width and the height of each rectangle will be proportional to frequency (no. of workers).
Note : Since the first class interval is start from 80 not from 0 , hence we use a
'kink' or a break on the X -axis.

### 9.5.4 Pie Chart or Circle Graph

A pie chart is a way of showing how something is shared or divided. It shows the relationship between a whole and its parts.
In pie charts, entire circle of appropriate radius represent total allocation. The circle is divided into sectors. The size of the each sector is proportional to the information it represents.

Illustration 5
The way Mr. Goel spends his allowance is given below. Draw the pie chart.

| Iteam | Percent |
| :--- | :---: |
| Hobby | $10 \%$ |
| Lunch | $20 \%$ |
| Recreation | $45 \%$ |
| Savings | $25 \%$ |
| Total | $100 \%$ |

Solution

| Iteam | Percent | Fractional Part | Central Angle |
| :---: | :---: | :---: | :---: |
| Hobby | $10 \%$ | $\frac{10}{100}=\frac{1}{10}$ | $\frac{1}{10} \times 360^{\circ}=36^{\circ}$ |
| Lunch | $20 \%$ | $\frac{20}{100}=\frac{1}{5}$ | $\frac{1}{5} \times 360^{\circ}=72^{\circ}$ |
| Recreation | $45 \%$ | $\frac{45}{100}=\frac{9}{20}$ | $\frac{9}{20} \times 360^{\circ}=162^{\circ}$ |
| Savings | $25 \%$ | $\frac{25}{100}=\frac{1}{4}$ | $\frac{1}{4} \times 360^{\circ}=90^{\circ}$ |



### 9.5.5 Double Bar Graph

A double bar graph shows two sets of data simultaneously. It is useful for comparison of data.


## Illustration 6

Read the graph and answer the following questions:

(a) What is the information given by double bar graph?
(b) Which player has secured the maximum runs and he belongs to which team?
(c) What is the difference between runs scored by player 2 of two teams?
(d) Which team has scored more runs in case of fourth player?

## Solution

(a) Runs scored by 4 players of two teams.
(b) Player 3 of Team A has scored the maximum runs.
(c) $30-10=20$. Difference between runs scored by player 2 of both the teams.
(d) Player 4 of team B has scored more runs.

- Presentation of data in classes alongwith the corresponding frequencies of the classes is called grouped frequency distribution.
- In the class interval say 10-20, 10 is called the lower limit and 20 the upper limit.
- Smaller the class interval, more accurate are the results.
- Mid point of a class is called its class mark.
- Histograms are drawn for continuous class intervals.
- Histograms differ from the bar graphs in the fact that in histograms there are no gaps between two adjacent rectangles whereas in bar graphs, there are uniform gaps between the two adjacent rectangles.
- Circular diagram representing the data by means of sectors of a circle is called a pie chart.
- To draw a pie chart, we construct a circle and divide it into sectors proportional to the numerical values of the items given in the data.
- In a bar graph, bars of uniform width are drawn with various heights. The height of the bar represents the frequency of the corresponding observation.
- The difference between the minimum and maximum values of the observation is called the range of the data.
- Class-intervals of the type 1-5, 6-10, 11-15, ....... are called discontinuous class-intervals. Here the upper limit is included.
- Class-intervals of the type 1-5,6-10,11-15,....... are called continuous class-intervals. Here the upper limit is excluded.
- The difference between the upper and lower class limits of an interval is called width or size of the interval.
- The mid-point of a class-interval is called its class-mark or mid-value.
- Histogram is a graphical representation of a grouped frequency distribution in the form of rectangles of equal width with class-intervals as bases and the corresponding frequencies as heights, there being no gap between any two successive rectangles.
- In a pie-chart, the values of different components are represented by the sectors of a circle. The total angle of $360^{\circ}$ at the centre of a circle is divided according to the values of the components.

Central angle for a component $=\frac{\text { Value of the component }}{\text { Total value }} \times 360^{\circ}$

## SOLVED EXAMPLE

## Example 1 :

The bar graph given in figure represents the class-wise enrollment of a secondary school. Read the graph and answer the following questions.
(a) Which class has the maximum number of students and what is the number?
(b) In which class the number of boys equals the number of girls?
(c) Find the ratio of the number of boys to girls in class VIII.
(d) What is the difference between the number of boys and the number of girls in class $\mathbf{X}$ ?
(e) What is the enrollment of class VII ?


## Solution :

(a) Class VI Number of boys $=30$

Number of girls $=25$
Total students $=30+25=55$
(b) Class IX
(c) Boys : girls $=20: 22$ or $10: 11$
(d) Number of boys $=24$

Number of girls = 19
Difference $=24-19=5$
(e) Number of boys $=23$

Number of girls $=17$
Total enrollment $=23+17=40$

## Example 2:

The frequency distribution of weights (in $\mathbf{k g}$ ) of $\mathbf{4 0}$ persons of a locality is given below

| Weight(in kg) | $40-45$ | $45-50$ | $50-55$ | $55-60$ | $60-65$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| No. of persons | 4 | 12 | 13 | 6 | 5 |

(a) What is the upper limit of the first class interval?
(b) Find the class marks of the second class interval?
(c) What is the class size of each class?
(d) Which class interval has the highest frequency?
(e) What is the lower limit of the last class interval?

Solution :
(a) Upper limit of the first class interval $(40-45)$ is 45 .
(b) Class marks of the second class interval $(45-50)$ is $\frac{45+50}{2}=\frac{95}{2}=47.5$
(c) Class size of each class is 5 .
(d) Class (50-55) has the highest frequency (13).
(e) Lower limit of the last class interval ( $60-65$ ) is 60 .

## Example 3 :

The average temperature ( ${ }^{\circ} \mathrm{C}$ ) of a city for the month of January, 2010 are given below :
9.3, 10.5, 9.3, 10.4, 9.0, 5.1, 11.2, 10.5, 7.1, 8.3, 10.0, 11.0, 12.5, 12.2, 12.0, 11.9, 7.6, 8.9, 9.2, 9.0, 5.6, 7.8, 6.9, 7.5, 8.5, 8.6, 6.3, 7.0, 8.0, 7.1, 5.7.

Prepare grouped frequency table using equal class size taking one of the classes as $7 \mathbf{- 8}$.
(a) What was the minimum temperature?
(b) What was the maximum temperature?

Solution :
(a) The minimum temperature was $5.1^{\circ} \mathrm{C}$.
(b) The maximum temperature was $12.5^{\circ} \mathrm{C}$.

One of the class intervals is $7-8$ and class size is 1 .
$\therefore \quad$ The class intervals will be $5-6,6-7,7-8,8-9,9-10,10-11,11-12,12-13$.
Grouped Frequency Distribution of Temperature


## Example 4 :

The height of 20 students of a class are given as under

| Heigths(in cm) | Number of students |
| :---: | :---: |
| $120-125$ | 1 |
| $125-130$ | 2 |
| $130-135$ | 4 |
| $135-140$ | 2 |
| $140-145$ | 6 |
| $145-150$ | 4 |
| $150-155$ | 1 |

Draw a bar graph to represent the above data.

## Solution :

Take along horizontal axis the heights and along vertical axis the frequencies. We represent each class interval by 10 small divisions along horizontal axis and take heights proportional to the corresponding frequencies. Since the scale on horizontal axis starts at 120, a kink (break) is indicated near the origin to show that the graph is drawn to scale beginning from 120 and not from the origin.
The bar graph is shown in figure.


## Example 5:

A survey was made to find the types of mu sic that a certain group of young people linked in a city. The pie chart in figure depicts the findings of this survey. From this pie chart answer the following questions
(a) In 20 people like semi-classical music, how many young people were surveyed?
(b) Which type of music is liked by the maximum number of people?

(c) If a cassette company were to make 1000 CDs , how many of each type would they make?

## Solution :

(a) $20 \%$ of the total people surveyed $=20$

$$
\therefore \quad \text { Total people surveyed }=\frac{20 \times 100}{20}=100
$$

(b) Maximum percentage $=40 \%$
$\therefore \quad$ Light music is liked by maximum number of people.
(c) CDs of light music $=40 \%$ of $1000=\frac{40 \times 1000}{100}=400$

CDs of folk music $=30 \%$ of $1000=\frac{30 \times 1000}{100}=300$
CDs of classical music $=10 \%$ of $1000=\frac{10 \times 1000}{100}=100$
CDs of semi-classical music $=20 \%$ of $1000=\frac{20 \times 1000}{100}=200$

## Example 6:

Pooja spends different hours of a working day as follows :

| Activity | School | Home assignments | Play | Sleep | Other |
| :---: | :---: | :---: | :---: | :---: | :---: |
| No. of hours | 8 | 3 | 2 | 8 | 3 |

## Draw a pie chart to represent the above data.

## Solution :

We know that central angle for a variable $=\frac{\text { Frequency of the variable }}{\text { Total of frequencies }} \times 360^{\circ}$
Total frequency $=8+3+2+8+3=24$
$\therefore \quad$ Central angle for school $=\frac{8}{24} \times 360^{\circ}=120^{\circ}$
Central angle for home assignment $=\frac{3}{24} \times 360^{\circ}=45^{\circ}$
Central angle for play $=\frac{2}{24} \times 360^{\circ}=30^{\circ}$


Central angle for sleep $=\frac{8}{24} \times 360^{\circ}=120^{\circ}$
Central angle for other activities $=\frac{3}{24} \times 360^{\circ}=45^{\circ}$
Now we draw a pie chart as shown in figure.

## Example 7:

The sale of car batteries of a company per day in a particular month is given below :

| $\mathbf{6 6}$ | $\mathbf{7 6}$ | $\mathbf{8 5}$ | $\mathbf{4 5}$ | $\mathbf{8 7}$ | $\mathbf{9 8}$ | $\mathbf{6 5}$ | $\mathbf{8 2}$ | $\mathbf{8 5}$ | $\mathbf{7 4}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{8 2}$ | $\mathbf{4 7}$ | $\mathbf{9 6}$ | $\mathbf{1 0 0}$ | $\mathbf{5 0}$ | $\mathbf{1 1 0}$ | $\mathbf{8 1}$ | $\mathbf{7 8}$ | $\mathbf{5 2}$ | $\mathbf{4 7}$ |
| $\mathbf{4 8}$ | $\mathbf{7 2}$ | $\mathbf{6 5}$ | $\mathbf{5 8}$ | $\mathbf{8 5}$ | $\mathbf{9 5}$ | $\mathbf{1 0 8}$ | $\mathbf{6 8}$ | $\mathbf{5 8}$ | $\mathbf{7 2}$ |

From the frequencies taking one of the class intervals as $\mathbf{8 0 - 9 0}$.

Solution :

| Class Intervals | Tally marks | Frequency |
| :---: | :---: | :---: |
| $40-50$ | $\\|\\|\\|$ | 4 |
| $50-60$ | $\\|\\|\\|$ | 4 |
| $60-70$ | $\\|\\|\\|$ | 4 |
| $70-80$ | $\mid \mathbb{}$ | 5 |
| $80-90$ | $\|W\| \mid$ | 7 |
| $90-100$ | $\|\|\mid$ | 3 |
| $100-110$ | $\\|$ | 2 |
| $110-120$ | $\mid$ | 1 |
| Total |  | 30 |

## Example 8 :

Read the graph and answer the following questions :

(a) What is the information given by double bar graph ?
(b) Which player has secured the maximum runs and he belongs to which team?
(c) What is the difference between runs scored by player 2 of two teams?
(d) Which team has scored more runs in case of fourth player?

## Solution :

(a) Runs scored by 4 players of two teams.
(b) Player 3 of Team $A$ has scored the maximum runs.
(c) $30-10=20$. Difference between runs scored by player 2 of both the teams.
(d) Player 4 of team $B$ has scored more runs.

## Example 9 :

The following data represents rainfall (in cm ) in 30 days in a month :

| 25 | 19 | 12 | 10 | 16 | 2 | 13 | 7 | 22 | 29 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 14 | 13 | 9 | 8 | 13 | 6 | 26 | 1 | 15 | 19 |
| 17 | 12 | 8 | 4 | 3 | 2 | 10 | 15 | 20 |  |

(a) Form a frequency distribution table taking one of the classes as 15-20.
(b) What is the upper limit of the class 10-15?
(c) What is the class mark of the class 15-20?
(d) What is the frequency corresponding to the class 20-25 ?

## Solution :

(a) Frequency Distribution Table:

| Class Intervals | Tally marks | Frequency |
| :---: | :---: | :---: |
| $0-5$ | $\|\|\|\mid$ | 4 |
| $5-10$ | $\mathbb{X}$ | 5 |
| $10-15$ | $\mathbb{X}\|\|\mid$ | 8 |
| $15-20$ | $\mathbb{X} \mid$ | 6 |
| $20-25$ | $\|\|\mid$ | 3 |
| $25-30$ | $\\|\\|$ | 4 |
| Total |  | 30 |

(b) Upper limit of class $10-15$ is 15 .
(c) Class mark of class $15-20=\frac{15+20}{2}=\frac{35}{2}=18.5$
(d) Frequency corresponding to the class $20-25$ is 3 .

## Example 10:

Read the graph given in figure and answer the questions that follow the graph :

(a) In which part of educational system was the maximum provision made?
(b) What was the total amount of money estimated for smooth functioning of the educational system of the country?
(c) What percentage of total estimated amount was kepted for training of teachers?
(d) Under which head of least provision of estimated amount was made?

Solution :
(a) Primary schools
(b) $26+18+16+8+4+14+6+18=110$ crore
(c) Estimated amount provided for training of teachers $=$ Rs. 6 crore

Total estimated amount $=$ Rs. 110 crore

$$
\therefore \quad \text { Required percentage }=\frac{6}{110} \times 110 \%=\frac{60}{11} \%=5 \frac{5}{11} \%
$$

(d) Mid-day meal.

## Example 11 :

Histogram for the daily expenses of a group of tourists is given in the frequency. Read the histogram and construct the corresponding grouped frequency distribution table. Also answer the following questions :

(a) What information is depicted by the histogram?
(b) What is the highest daily expenses?
(c) Find the average expenditure of the group which has spent the maximum.
(d) How many tourists spent on an average Rs. 250 ?
(e) What is the total number of tourists?
(f) Which groups contain the same number of tourists?

## Solution :

The required grouped frequency table is given below :
(a) The histogram depicts daily expenses (in Rs.) of a group of tourists.
(b) Rs. 600
(c) Class interval having maximum expenditure is $500-600$.

$$
\therefore \quad \text { Average expenditure of this group }=\frac{500+600}{2}=\text { Rs. } 550
$$

(d) Average of 200 and 300 is $\frac{200+300}{2}=$ Rs. 250 and the number of tourists in the class interval $200-300$ is 60 .
Thus, number of tourists with an average expenditure of Rs. 250 is 60 .
(e) Total number of tourists $=30+40+60+40+10+20=200$
(f) Groups having same number of tourists are 100-200 and 300-400.

## Example 12 :

The following distribution gives the weights (in $\mathbf{k g}$ ) of 80 persons :

| Years | $30-35$ | $35-40$ | $40-45$ | $45-50$ | $50-55$ | $55-60$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of Students | 20 | 25 | 15 | 5 | 10 | 5 |

## Draw a histogram for the above data.

## Solution :

Here class intervals showing the weights of the persons are taken against X -axis and the number of persons along Y-axis. As we have to start from 30(and not from 0), a kink (break) is taken before the class interval $30-35$ on X-axis. It indicates that the distance from 0 to 30 has not been shown.


## Example 13 :

The percentage of expendicutre of a company under different heads is given below :

| Heads of Expendicutre | Percentage |
| :---: | :---: |
| Salary | 40 |
| Conveyance | 10 |
| Machines | 25 |
| Rent | 20 |
| Others | 5 |

Represent the above information by means of a pie chart.

## Solution :

Table for calculation for construction pie chart :

| Heads of Expendicutre | Percentage | Central angle |
| :---: | :---: | :---: |
| Salary | 40 | $\frac{40}{100} \times 360^{\circ}=144^{\circ}$ |
| Conveyance | 10 | $\frac{10}{100} \times 360^{\circ}=36^{\circ}$ |
| Machines | 25 | $\frac{25}{100} \times 360^{\circ}=90^{\circ}$ |
| Rent | 20 | $\frac{20}{100} \times 360^{\circ}=72^{\circ}$ |
| Others | 5 | $\frac{5}{100} \times 360^{\circ}=18^{\circ}$ |
| Total | 100 | $360^{\circ}$ |

The required pie chart is drawn in figure.


## Example 14 :

The pie chart given in the figure, depicts the marks obtained by a student in different subjects out of a total of $\mathbf{4 5 0}$ marks. Find the marks obtained in each subject.

## Solution :

We shall calculate the marks obtained in each subject on the basis of the measure of the central angles of the sectors given in the pie chart.
Marks obtained in English $=\frac{72^{\circ}}{360^{\circ}} \times 450=90$
Marks obtained in Sanskrit $=\frac{44^{\circ}}{360^{\circ}} \times 450=55$
Marks obtained in Hindi $=\frac{52^{\circ}}{360^{\circ}} \times 450=65$
Marks obtained in Mathematics $=\frac{84^{\circ}}{360^{\circ}} \times 450=105$


Marks obtained in Gen. Science $=\frac{68^{\circ}}{360^{\circ}} \times 450=85$
Marks obtained in Soc. Studies $=\frac{40^{\circ}}{360^{\circ}} \times 450=50$

## Example 15:

The number of scooters produced by a factory in four years are 7200. Find the number of scooters obtained in each year.


## Solution :

Scooters produced in $2002=\frac{100^{\circ}}{360^{\circ}} \times 7200=2000$
Scooters produced in $2003=\frac{120^{\circ}}{360^{\circ}} \times 7200=2400$
Scooters produced in $2004=\frac{45^{\circ}}{360^{\circ}} \times 7200=900$
Scooters produced in $2005=\frac{95^{\circ}}{360^{\circ}} \times 7200=1900$

## Example 16 :

The following bar graph shows the after school activity preferred by the students of a school. Study the bar graph and answer the following questions :

(i) What is the scale along the horizontal axis ?
(ii) How many students are considered ?
(iii) How many students preferred outdoor games?
(iv) Which activity is preferred by maximum number of students?
(v) Which activity is preferred by least number of students?
(vi) How many more students played video games than played indoor games?
(vii) How many students do not play any kind of game?

## Solution :

(i) The scale along horizontal axis is 10 small divisions $=10$ students
or 1 small division $=1$ student
(ii) Number of students preferring various activities are : Indoor games : 30 , Outdoor games : 40, TV:20, Reading : 25 , Video games : 70 $\therefore \quad$ total number of students considered $=30+40+20+25+70=185$
(iii) Number of students preferring outdoor games $=40$
(iv) The length of the bar corresponding to video games is largest, so video games are preferred by maximum number of students.
(v) The bar corresponding to TV shortest, so TV is preferred by minimum number of students.
(vi) Number of students who preferred video games $=70$

Number of students who preferred indoor games $=30$
$\therefore \quad$ required number of students $=70-30=40$
(viii) Out of the total number of students considered, who do not play any kind of game, preferred either reading or TV.
$\therefore \quad$ Required number of students $=$ students preferred TV + students preferred reading

$$
=20+25=45 .
$$

## Example 17 :

Study the double bar graph given below and answer the questions that follow :

(i) What is the information given by the double bar graph?
(ii) In which subject has the performance improved the most?
(iii) In which subject has the performance deteriorated?
(iv) In which subject is the performance at par?

## Solution :

(i) The given double bar graph shows the marks obtained by a student in 5 subjects in the year 2006-07 and 2007-08.
(ii) From the graph, we see that the difference in heights of bars representing Maths for 2006-07 and 2007-08 is maximum. Hence, in Maths the performance has improved most.
(iii) Look at the bars representing English. We see that the bar for the year 2007-08 is shorter than the bar for the year 2006-07. Hence, in English, the performance has deteriorated.
(iv) We see that the heights of both the bars representing Hind is same. Hence, in Hindi the performance is at par.

## Example 18 :

The following are the daily rents (in rupees) of 40 shops :
$45,50,60,53,55,51,32,35,40,42,80,82,65,63,62,45,50,51,32,51,82,89,35,45,80,65$, $63,32,55,53,45,83,80,75,74,79,71,72,75,32$.
Construct a grouped frequency table for the above data.

## Solution :

Least rent $=$ Rs. 32, Greatest rent $=$ Rs. 89
Range of the raw data $=89-32=57$
Since the rents lie between 32 to 89 , it will be easy to divide the range in 6 classes of equal size.
Size of class-intervals $=$ Range $\div$ No. of classes

$$
=57 \div 6=9.5 \sim 10
$$

Hence, the class intervals covering the data are :
$30-40,40-50,50-60,60-70,60-70,70-80,80-90$

| Class Intervals | Tally marks | Frequency |
| :---: | :---: | :---: |
| $30-40$ | $\mathbb{X} \mid$ | 6 |
| $40-50$ | $\mathbb{X} \mid$ | 6 |
| $50-60$ | $\mathbb{X}\|\|\|\mid$ | 9 |
| $60-70$ | $\mathbb{X} \mid$ | 6 |
| $70-80$ | $\mathbb{X} \mid$ | 6 |
| $80-90$ | $\mathbb{X}\|\mid$ | 7 |
|  |  | 40 |

## Example 19 :

Given below is the histogram depicting the age (in years) of school teachers. Read the histogram and answer the questions that follow :
There is a $\bigwedge($ kink ) before the class-interval 20-25 on the horizontal axis. It shows that the full distance 0 to 20 is not shown.

(i) What is the class-size?
(ii) How many teachers are there in the age group 30-35 ?
(iii) In which age groups the number of teachers are maximum?
(iv) How many teachers in all are considered ?
(v) How many teachers are of below 40 years?

## Solution :

(i) Class-size $=$ Upper limit - Lower limit (of any class)

$$
=30-25=40-45=5
$$

(ii) In the class-interval $30-35$, there are 10 teachers.
(iii) In $40-45$ age group, the number of teachers are maximum.
(iv) $4+8+10+6+16+12+8=64$ teachers are considered in all.
(v) $4+8+10+6=28$.

## Example 20 :

The weekly wages (in Rs.) of $\mathbf{3 0}$ workers in a factory are :
$830,835,890,810,835,836,869,845,898,890,820,860,832,833,855,845,804,808,812$,
$840,885,835,836,878,840,868,840,890,806,840$
Using tally marks, make a frequency table with intervals $800-810,810-820$ and so on.
Also, draw a histogram for the above frequency table and the answer the following questions :
(i) Which group has maximum number of workers?
(ii) How many workers earn Rs. 850 and more?
(iii) How many workers earn less than Rs. 850?

## Solution :

We make group of observations say $800-810,810-820$, etc. and obtain a frequency distribution of the number of observations falling in each group. Thus, the frequency distribution table for the given data can be as shown below :


We represents the group of observations (class - intervals) along the horizontal axis on a suitable scale and the frequencies along the vertical axis on a suitable scale. Taking class intervals as bases and the corresponding frequencies as heights, we construct rectangles to obtain the histogram of the above frequency distribution as shown. Broken line of jagged line has been used along the horizontal line to indicate that we are not showing the numbers between 0 to 800 .

(i) The maximum number of workers are in the group $830-840$.
(ii) Required number of workers $=1+3+1+1+4=10$
(iii) Required number of workers $=3+2+1+9+5=20$.

## Example 21 :

The eye colour of 60 people is recorded as below :

| Eye colour | Blue | Grey | Brown | Hazel | Total |
| :---: | :---: | :---: | :---: | :---: | :---: |
| No. of people | 12 | 10 | 25 | 13 | 60 |

Draw a pie chart to represent the above data.

## Solution :

To draw a pie chart, the first step is to calculate the angles at the centre of the circle. The total angle of $360^{\circ}$ at the centre of a circle is divided according to the values of the components.

Central angle for a component $=\frac{\text { Value of the component }}{\text { Total value }} \times 360^{\circ}$
So, Central angle for blue $=\frac{12}{60} \times 360^{\circ}=72^{\circ}$
Central angle for grey $=\frac{10}{60} \times 360^{\circ}=60^{\circ}$
Central angle for brown $=\frac{25}{60} \times 360^{\circ}=150^{\circ}$


Central angle for hazel $=\frac{13}{60} \times 360^{\circ}=78^{\circ}$
Now we draw a circle of a convenient radius and divide it into sectors of angles $72^{\circ}, 60^{\circ}, 150^{\circ}$ and $78^{\circ}$.

## Example 22 :

The following table shows the subjects preferred by 300 students during a survey.

| Subjects | No. of students |
| :---: | :---: |
| Maths | 75 |
| Science | 60 |
| English | 90 |
| Moral Science | 40 |
| Social Science | 35 |
| Total | 300 |

Draw a pie chart to represent the above data.

## Solution :

We first find the central angles for various components.
Central angle for a component $=\frac{\text { Value of the component }}{\text { Total value }} \times 360^{\circ}$
Central angle for Maths $=\frac{75}{300} \times 360^{\circ}=90^{\circ}$
Central angle for Science $=\frac{60}{300} \times 360^{\circ}=72^{\circ}$
Central angle for English $=\frac{90}{300} \times 360^{\circ}=108^{\circ}$
Central angle for Moral Science $=\frac{40}{300} \times 360^{\circ}=48^{\circ}$


Central angle for Social Science $=\frac{35}{300} \times 360^{\circ}=42^{\circ}$
Now, we draw a circle of a convenient radius and divide it into sectors of angles $90^{\circ}, 72^{\circ}, 108^{\circ}, 48^{\circ}$ and $42^{\circ}$.

## Example 23 :

Monthly income of a family is Rs. 9000 . The following pie chart shows how the family used the money.
Study the pie-chart and answer the following questions :
(i) How much was spent on food?
(ii) How much was spent on education?
(iii) How much was spent on rent?
(iv) How much does the family save?
(v) Find the ratio of the amount spent on other items to that
 spent on food.

## Solution :

Total income of the family=Rs. 9000
Value of a component $=\frac{\text { Central angle for the component }}{360} \times$ total value
(i) Amount spent on food $=$ Rs. $\frac{80}{360} \times 9000=$ Rs. 2000
(ii) Amount spent on education $=$ Rs. $\frac{50}{360} \times 9000=$ Rs. 1250
(iii) Amount spent on rent $=$ Rs. $\frac{100}{360} \times 9000=$ Rs. 2500
(iv) Savings of the family $=$ Rs. $\frac{40}{360} \times 9000=$ Rs. 1000
(v) Amount spent on other items $=$ Rs. $\frac{90}{360} \times 9000=$ Rs. 2250

From (i), amount spent on food = Rs. 2000
$\therefore \quad$ Required ratio $=2250: 2000=9: 8$.

## CONCEPT APPLICATION LEVEL - I <br> [NCERT Questions]

## EXERCISE 1

Q. 1 For which of these would you use a histogram to show the data?
(a) The number of letters for different areas in a postman's bag.
(b) The height of competitors in an athletics meet.
(c) The number of cassettes produced by 5 companies.
(d) The number of passengers boarding trains from 7:00 a.m. to 7:00 p.m. at a station. Given reasons for each.
Sol. For (b) and (d) because in these two cases data can be divided into class intervals.
Q.2. The shoppers who come to a departmental store are marked as : man (M), woman (W), boy (B) or girl (G). The following list gives the shoppers who came during the first hour in the morning.
WWWGBWWMGGMMWWWWGBMWBGGMWWMMWWWMWBW GMWWWWGWMMWWMWGWMGWMMBGGW
Make a frequency distribution table using tally marks. Draw a bar graph to illustrate it.

Sol.


| Types of Shoppers | Tally Marks | Number of Shoppers |
| :---: | :---: | :---: |
| W | $\mathbb{N} \mathbb{N} \mathbb{N} \mathbb{N} \mathbb{N}$ II\\| | 28 |
| M | $\mathbb{N} \mathbb{N} \mathbb{N}$ | 15 |
| B | $\mathbb{N}$ | 5 |
| G | $\mathbb{N} \mathbb{N} \\|$ | 12 |

Q.3. The weekly wages (in ₹) of $\mathbf{3 0}$ workers in a factory are :
$830,835,890,810,835,836,869,845,898,890,820,860,832,833,855,845,804,808,812$, 840, 885, 835, 835, 836, 878, 840, 868, 890, 806, 840.
Using tally marks make a frequency table with intervals as $800-810,810-820$ and so.

Sol.

Q. 4 Draw a histogram for the frequency table made for the data in Question 3, and answer the following questions :
(i) Which group has the maximum number of workers?
(ii) How many workers earn ₹ 850 and more?
(iii) How many workers earn less than ₹ 850 ?

(i) Group 830-840, has the maximum number of workers.
(ii) $1+3+1+1+4=10$, workers earn ₹ 850 and more.
(iii) $3+2+1+9+5=20$, workers earn less than ₹ 850 .
Q. 5 The number of hours for which students of a particular class watched television during holidays is shown through the given graph :


Answer the following :
(i) For how many hours did the maximum number of students watch TV?
(ii) How many students watched TV for less than 4 hours?
(iii) How many students spent more than 5 hours in watching TV?

Sol. (i) The maximum number of students watch TV for 4 to 5 hours.
(ii) $4+8+22=34$ students watch TV for less than 4 hours.
(iii) $8+6=14$ students spend more than 5 hours in watching TV.

## EXERCISE - 2

Q. 1 A survey was made to find the type of music that a certain group of young people liked in a city. Adjoining pie chart shows the findings of this survey.


From this pie chart answer the following :
(i) If 20 people liked classical music, how many young people were surveyed?
(ii) Which type of music is liked by the maximum number of people?
(iii) If a cassette company were to make 1000 CD 's, how many of each type would they make?

Sol. (i) Suppose that x young people were surveyed. Then, the number of young people who liked classical music $=10 \%$ of x

$$
=\mathrm{x} \times \frac{10}{100}=\frac{\mathrm{x}}{10}
$$

According to the question :

$$
\frac{x}{10}=20 \quad \Rightarrow \quad x=20 \times 10 \quad \Rightarrow \quad x=200
$$



Hence, 200 young people were surveyed.
(ii) Light music liked by the maximum number of people.
(iii) Number of CD's of semi classical music $=20 \%$ of $1000=1000 \times \frac{20}{100}=200$

Number of CD's of classical music $=10 \%$ of $1000=1000 \times \frac{10}{100}=100$
Number of CD's of Folk music $=30 \%$ of $1000=1000 \times \frac{30}{100}=300$
Number of CD's of light music $=40 \%$ of $1000=1000 \times \frac{40}{100}=400$
Q. 2 A group of $\mathbf{3 6 0}$ people were asked to vote for their favourite season from the three seasons rainy, winter and summer.

| Season | No. of Votes |
| :---: | :---: |
| Summer齿北 | 90 |
| Rainy | 120 |
| Winter | 150 |

(i) Which season got the most votes?
(ii) Find the central angle of each sector.
(iii) Draw a pie chart to show this information

Sol. (i) Winter season got the most votes.
(ii) Central angle of winter sector $=\frac{\text { Number of people who vote for winter season }}{\text { Total number of people }} \times 360^{\circ}$

$$
=\frac{150}{360} \times 360^{\circ}=150^{\circ}
$$

(iii)

| Season | People who <br> Voted for it | In Fraction | Central angle |
| :---: | :---: | :---: | :---: |
| Winter | 150 | $\frac{5}{12}$ | $\frac{5}{12}$ of $360^{\circ}=150^{\circ}$ |
| Summer | 90 | $\frac{1}{4}$ | $\frac{5}{12}$ of $360^{\circ}=90^{\circ}$ |
| Rainy | 120 | $\frac{1}{3}$ | $\frac{5}{12}$ of $360^{\circ}=120^{\circ}$ |



Central angle of summer sector $=\frac{\text { Number of people who vote for summer season }}{\text { Total number of people }} \times 360^{\circ}$

$$
=\frac{90}{360} \times 360^{\circ}=90^{\circ}
$$

$\begin{aligned} \text { Central angle of rainy sector } & =\frac{\text { Number of people who vote for rainy season }}{\text { Total number of people }} \times 360^{\circ} \\ & =\frac{120}{360} \times 360^{\circ}=120^{\circ}\end{aligned}$
Q. 3 Draw a pie chart showing the following information. The table shows the colours prefered by a group of people.

| Colours | Number of <br> people |
| :--- | :---: |
| Blue | 18 |
| Green | 9 |
| Red | 6 |
| Yellow | 3 |
| Total | $\mathbf{3 6}$ |

Find the proportion of each sector. For example, Blue is
$\frac{18}{36}=\frac{1}{2} \quad ;$ Green is $\frac{9}{36}=\frac{1}{4}$ and so on.
Use this to find the corresponding angles.

Sol.


| Colours | Number of People | Proportion | Corresponding Angle |
| :---: | :---: | :---: | :---: |
| Blue | 18 | $\frac{18}{36}=\frac{1}{2}$ | $\frac{1}{2} \times 360^{\circ}=180^{\circ}$ |
| Green | 9 | $\frac{9}{36}=\frac{1}{4}$ | $\frac{1}{4} \times 360^{\circ}=90^{\circ}$ |
| Red | 6 | $\frac{6}{36}=\frac{1}{6}$ | $\frac{1}{6} \times 360^{\circ}=60^{\circ}$ |
| Yellow | 3 | $\frac{3}{36}=\frac{1}{12}$ | $\frac{1}{12} \times 360^{\circ}=30^{\circ}$ |
| Total | 36 | 1 | $360^{\circ}$ |

Q. 4 The adjoining pie chart gives the marks scored in an examination by a student in Hindi, English, Mathematics, Social Science and Science. If the total marks obtained by the student were 540, answer the following questions.
(i) In which subject did the student score 105 marks?
(Hint : For 540 marks, the central angle $=360^{\circ}$. So, for 105 marks, what is the central angle?)
(ii) How many more marks were obtained by the student in Mathematics than in Hindi?
(iii) Examine whether the sum of the marks obtained in Social Science and Mathematics is more than that in Science and Hindi.
(Hint : Just study the central angles.)
Sol. (i) $\quad \therefore \quad$ For 540 marks, the central angle $=360^{\circ}$
$\therefore \quad$ For 105 marks, the central angle $=\frac{360^{\circ}}{540^{\circ}} \times 105=70^{\circ}$
Hence, the student scored 105 marks in Hindi.
(ii) Marks obtained by the student in Mathematics $=\frac{90^{\circ}}{360^{\circ}} \times 540=135$

Marks obtained by the student in Hindi $=105$
Hence, the student obtained 135-105 = 30 marks more in Mathematics than in Hindi
(iii) Sum of the central angles for Social Science and Mathematics $=65^{\circ}+90^{\circ}=155^{\circ}$

Sum of the central angles for Science and Hindi $=80^{\circ}+70^{\circ}=150^{\circ}$
Hence, yes! the sum of the marks obtained in Social Science and Mathematics is more than in Science and Hindi.
Q. 5 The number of students in a hostel, Speaking different languages is given below display the data in a pie chart.

| Language | Hindi | English | Marathi | Tamil | Bengali | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number <br> of students | 40 | 12 | 9 | 7 | 4 | 72 |

Sol.

| Language | Number of Students | In Fraction | Central angle |
| :---: | :---: | :---: | :---: |
| Hindi | 40 | $\frac{40}{72}=\frac{5}{9}$ | $\frac{5}{9} \times 360^{\circ}=200^{\circ}$ |
| English | 12 | $\frac{12}{72}=\frac{1}{6}$ | $\frac{1}{6} \times 360^{\circ}=60^{\circ}$ |
| Marathi | 9 | $\frac{9}{72}=\frac{1}{8}$ | $\frac{1}{8} \times 360^{\circ}=45^{\circ}$ |
| Tamil | 7 | $\frac{7}{72}$ | $\frac{7}{72} \times 360^{\circ}=35^{\circ}$ |
| Bengali | 4 | $\frac{4}{72}=\frac{1}{18}$ | $\frac{1}{18} \times 360^{\circ}=20^{\circ}$ |
| Total | 72 | 1 | $360^{\circ}$ |

Now, we make the pie chart.

## EXERCISE 3

Q. 1 List the outcomes you can see in these experiments.
(a) spinning a wheel

(b) Tossing two coins together

Sol. (a) A, B, C and D.
(b) HT, HH, TH, TT (Here HT means Head on first coin and Tail on the second coin and so on)
Q. 2 When a die is thrown, list the outcomes of an event of getting.
(i) (a) a prime number
(b) not a prime number
(ii) (a) a number greater than 5
(b) a number not greater than 5.

Sol. (i)
(a) $2,3,5$
(b) $1,4,6$
(a) 6
(b) $1,2,3,4,5$
(ii)
Q. 3 Find the
(a) Probability of the pointer stopping of $D$ in (question 1(a))?
(b) Probability of getting an ace from the well shuffled deck of 52 playing cards?
(c) Probability of getting a red apply (see figure)


Sol. (a) There are in all 5 outcomes of the event. The pointer stopping on D has only 1 outcome $\therefore \quad$ Probability of the pointer stopping on $D=\frac{1}{5}$
(b) Probability of getting an ace from well shuffled deck of 52 playing cards $=\frac{4}{52}=\frac{1}{13}$
$\therefore \quad$ There are in all 52 card out of which 4 ace a card
(c) Probability of getting a red apple $=\frac{4}{7}$
$\therefore \quad$ There are in all 7 apples out of which 4 are red.
Q. 4 Numbers 1 to 10 are written on ten separate slips (one number on one slip), kept in a box and mixed well. One slip is chosen from the box without looking into it. What is the probability of.
(i) getting a number 6 ?
(ii) getting a number less than 6 ?
(iii) getting a number greater than 6 ?
(iv) getting a 1 -digit number?

Sol. (i) There are all 10 outcome only. So, the probability of getting a number $6=\frac{1}{10}$
(ii) Probability of getting a number less than $6=\frac{5}{10}=\frac{1}{2}$

Getting a number less than 6 has five outcomes as there are five numbers (1,2,3, 4 and 5 ) less than 6.
(iii) Probability of getting a number greater than $6=\frac{4}{10}=\frac{2}{5}$

Getting a number greater than 6 has four outcomes as there are four numbers $(7,8,9,10)$ greater than 6 .
(iv) Probability of getting a 1-digit number $=\frac{9}{10}$

Getting a 1 -digit number has nine outcomes as there are nine number (1,2,3,4,5,6,7,8 and 9 ) of 1-digit.
Q. 5 If you have a spinning wheel with 3 green sectors, 1 blue sector and 1 red sector, what is the probability of getting a green sector? What is the probability of getting a non blue sector?
Sol. There are in all $3+1+1=5$ outcomes of the event.
$\therefore \quad$ Probability of getting a green sector.
Getting a green sector has 3 outcomes as there are in all 3 green sectors $=\frac{3}{5}$
Getting a non blue sector has $3+1=4$ outcomes as there are 3 green (non-blue) sectors and 1 red sector.
$\therefore \quad$ Probability of getting a non-blue sector $=\frac{3+1}{5}=\frac{4}{5}$

## Q. 6 Find the probabilities of the events given in questions 2.

Sol. There are in all 6 outcomes when a die is thrown.
(i) (a) getting a prime number consists of 3 outcomes only
$\therefore \quad$ Probability of getting a prime number $=\frac{3}{6}=\frac{1}{2}$
(i) (b) getting a non-prime number consists of 3 outcomes only
$\therefore \quad$ Probability of getting a non-prime number $=\frac{3}{6}=\frac{1}{2}$
(ii) (a) Getting a number greater than 5 has only one outcome.
$\therefore \quad$ Probability of getting a number greater than $5=\frac{1}{6}$
(ii) (b) Getting a number not greater than 5 has 5 outcomes only
$\therefore \quad$ Probability of getting a number not greater than $5=\frac{5}{6}$

## TRY THESE

Q. 1 Abargraph : A display of information using bars of uniform width, their height being proportional to the respective values

(i) What is the information given by the bar graph?
(ii) In which year is the increase in the number of students maximum?
(iii) In which year is the number of students maximum?
(iv) State whether true or false:
"The number of students during 2005-06 is twice that of 2003-04"
Sol. (i) The bar graphs gives the information about the number of students in class VIII in different academic years.
(ii) The increase in the number of students is maximum in the year 2004-05.
(iii) The number of students is maximum in the year 2007-08.
(iv) False
Q. 2 Double bar graph : it is a bar graph showing two sets of data simultaneously. It is useful for the comparison of data.

(i) What is the information given by the double bar graph?
(ii) In which subject has the performance improved the most?
(iii) In which subject has the performance deteriorated?
(iv) In which subject is the perfomance at par?

Sol. (i) The double bar graph gives the information about the marks obtained by a student in different subjects in the academic years 2005-06 and 2006-07.
(ii) The performance improved the most in the subject Mathematics.
(iii) The performance deteriorated in the subject English.
(iv) The performance is at par in the subject Hindi.

## Q. 3 Draw an appropriate graph to represent the given information.

| Month | July | August | September | October | November | December |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> watches sold | 1000 | 1500 | 1500 | 2000 | 2500 | 1500 |

Sol.

Q. 4

| Children who prefer | School A | School B | SchoolC |
| :---: | :---: | :---: | :---: |
| Walking | 40 | 55 | 15 |
| Cycling | 45 | 25 | 35 |

Sol.

Q. 5 Percentage wins in ODI by 8 top cricket teams.

| Teams | From Champions <br> Trophy to WorldCup-06 | Last 10 <br> ODI in 07 |
| :---: | :---: | :---: |
| South Africa | $75 \%$ | $78 \%$ |
| Australia | $61 \%$ | $40 \%$ |
| Sri Lanka | $54 \%$ | $38 \%$ |
| New Zealand | $47 \%$ | $50 \%$ |
| England | $46 \%$ | $50 \%$ |
| Pakistan | $45 \%$ | $44 \%$ |
| West Indies | $44 \%$ | $30 \%$ |
| India | $43 \%$ | $56 \%$ |


Q. 6 A group of students were asked to say which animal they would like most to have as a pet. The results are given below :
dog, cat, cat, fish, cat rabbit,dog, cat, rabbit, dog, cat, dog, dog, dog, cat, cow, fish, rabbit, dog, cat, dog, cat, cat, dog, rabbit, cat, fish, dog.
Make a frequency distribution table for the same.
Sol.

| Animal | Tally Marks | Number of Students |
| :---: | :---: | :---: |
| Dog | $\mathbb{N} \mathbb{N}$ | 10 |
| Cat | $\mathbb{N}$ | 10 |
| Fish | $\\|\\|$ | 3 |
| Rabbit | $\\|\\|\\|$ | 4 |
| Cow | $\mid$ | 1 |

Q. 7 Study the following frequency distribution table and answer the questions given below.

Frequency Distribution of Daily Income of 550 workers of a factory.
Table

| Class Interval (Daily <br> Income in Rupees) | Frequency (Number <br> of workers) |
| :---: | :---: |
| $100-125$ | 45 |
| $125-150$ | 25 |
| $150-175$ | 55 |
| $175-200$ | 125 |
| $200-225$ | 140 |
| $225-250$ | 55 |
| $250-275$ | 35 |
| $275-300$ | 50 |
| $300-325$ | 20 |
| Total | $\mathbf{5 5 0}$ |

Sol. (i) The size of the class interval is 25 .
(ii) The class 200-225 has the highest frequency.
(iii) The class 300-325 has the lowest frequency
(iv) The upper limit of the class interval 250-275 is 275 .
(v) The two classes 150-175 and 225-250 have the same frequency.
Q. 8 Construct a frequency distribution table for the data on weights (in $\mathbf{~ k g}$ ) of 20 students of a class using intervals 30-35, 35-40 and so on.
$40,38,33,48,60,53,31,46,34,36,49,41,55,49,65,42,44,47,38,39$.
Sol. Frequency distribution table for the data on weights (in kg ) of 20 students in a class.

| Class Interval [Weights <br> (in kg)] | Frequency (Number <br> of students) |
| :---: | :---: |
| $30-35$ | 3 |
| $35-40$ | 4 |
| $40-45$ | 4 |
| $45-50$ | 5 |
| $50-55$ | 1 |
| $55-60$ | 1 |
| $60-65$ | 1 |
| $65-70$ | 1 |
| Total | $\mathbf{2 0}$ |

Q. 9 Observe the histogram (Fig.) and answer the questions given below :


Fig.
(i) What information is being given by the histogram?
(ii) Which group contains maximum girls ?
(iii) How many girls have a height of 145 cm and more?
(iv) If we divide the girls into the following three categories, how many would there be in each? $\begin{array}{ll}150 \mathrm{~cm} \text { and more } & \text {-Group A } \\ 140 \mathrm{~cm} \text { to less than } 150 \mathrm{~cm} & \text {-Group B } \\ \text { Less than } 140 \mathrm{~cm} & \text {-Group C }\end{array}$
Less than 140 cm -Group C

Sol. (i) The graph gives the information about the heights in cm of girls of class VII.
(ii) The group 140-145 contains maximum girls.
(iii) $4+2+1=7$ girls have a height of 145 cm and more.
(iv) There would be $2+1=3$ girls in Group A.

There would be $7+4=11$ girls in Group B.
There would be $1+2+3=6$ girls in Group C.
Q. 10 Each of the following Pie charts (Fig. ) gives you a different piece of information about your class. Find the fraction of the circle representing each of these information.
(i)
(ii)
(iii)



Sol. (i) Proportion of the sector for girls $=\frac{50}{100}=\frac{1}{2}$
Proportion of the sector for boys $=\frac{50}{100}=\frac{1}{2}$
(ii) Proportion of the sector for walk $=\frac{40}{100}=\frac{2}{5}$

Proportion of the sector for cycle $=\frac{20}{100}=\frac{1}{5}$
Proportion of the sector for Bus or $\mathrm{Car}=\frac{40}{100}=\frac{2}{5}$
(iii) Proportion of the sector for Hate $=\frac{15}{100}=\frac{3}{20}$

Proportion of the sector for Love $=1-\frac{3}{20}$
$\because \quad$ The sum of all the fractions is $1=\frac{20-3}{20}=\frac{17}{20}$.
Q. 11 Answer the following questions based on the pie chart given (Fig.).
(i) Which type of programmes are viewed the most?
(ii) Which two types of programmes have number of viewers equal to those watching sports channels?
Sol. (i) Entertainment programmes are viewed the most.


Viewers watching different types of channels on T.V.

Fig.
(ii) News and Information types of programmes have number of viewers equal to those watching sports channels.
Q. 12 Draw a pie chart of the data given below. The time spent by a child during a day.

Sleep - 8 hours
School - 6 hours
Home work - 4 hours
Play - 4 hours
Others - 2 hours

Sol.

| Item | Hours | In Fraction | Central Angle |
| :---: | :---: | :---: | :---: |
| Sleep | 8 | $\frac{8}{24}=\frac{1}{3}$ | $\frac{1}{3} \times 360^{\circ}=120^{\circ}$ |
| School | 6 | $\frac{6}{24}=\frac{1}{4}$ | $\frac{1}{4} \times 360^{\circ}=90^{\circ}$ |
| Homework | 4 | $\frac{4}{24}=\frac{1}{6}$ | $\frac{1}{6} \times 360^{\circ}=60^{\circ}$ |
| Play | 4 | $\frac{4}{24}=\frac{1}{6}$ | $\frac{1}{6} \times 360^{\circ}=60^{\circ}$ |
| Others | 2 | $\frac{2}{24}=\frac{1}{12}$ | $\frac{1}{12} \times 360^{\circ}=30^{\circ}$ |



Now, we make the pie chart.
Which form of graph would be appropriate to display the following data.

## Q. 13 Production of food grains of a state.

| Year | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Production (in <br> lakh tons) | 60 | 50 | 70 | 55 | 80 | 85 |

Sol. Bar graph.

## Q. 14 Choice of food for a group of people.

| Favourite Food | Number of People |
| :---: | :---: |
| North Indian | 30 |
| South Indian | 40 |
| Chinese | 25 |
| Others | 25 |
| Total | $\mathbf{1 2 0}$ |

Sol. Bar graph.
Q. 15 The daily income of a group of a factory workers.

| Daily Income (in <br> Rupees) | Number of Workers (in <br> a Factory) |
| :---: | :---: |
| $75-100$ | 45 |
| $100-125$ | 35 |
| $125-150$ | 55 |
| $150-175$ | 30 |
| $175-200$ | 50 |
| $200-225$ | 125 |
| $225-250$ | 140 |
| Total | $\mathbf{4 8 0}$ |

Sol. Histogram.
Q. 16 When a die is thrown, what are the six possible outcomes?

Sol. 1,2,3,4,5 and 6
Q. 17 Suppose you spin the wheel.
(i) List the number of outcomes of getting a green sector and not getting a green sector on this wheel.

(ii) Find the probability of getting a green sector.
(iii) Find the probability of not getting a green sector.

Sol. (i) Number of outcomes of getting a green sector on this wheel $=5$
Number of outcomes of not getting a green sector on this wheel $=3$
(ii) Probability of getting a green sector $=\frac{5}{8}$

There are in all 8 outcomes of the event and the number of outcomes giving $G$ is 5 .
(iii) Probability of not getting a green sector $=\frac{3}{8}$

There are in all 8 outcomes of the event and the number of outcomes not giving G is 3 .

## CONCEPT APPLICATION LEVEL- II

Direction (Q. 1 to Q.5) :The graph in figure depicts the temperature recorded at difference times of a day of a particular city. Observe the graph and select the correct answer for the given four alternative answer
Q. 1 Maximum temperature was recorded at
(A) $5 \mathrm{a} . \mathrm{m}$.
(B) $7 \mathrm{a} . \mathrm{m}$.
(C) $9 \mathrm{a} . \mathrm{m}$.
(D) 1 p.m.
Q. $2 \quad 60^{\circ}$ temperature was recorded at
(A) 9 a.m.
(B) 1 p.m.
(C) 11 a.m.
(D) $9 \mathrm{p} . \mathrm{m}$.
Q. 3 The ratio of maximum temperature to minimum temperature is
(A) $5: 7$
(B) $13: 10$
(C) $12: 13$
(D) $7: 5$
Q. 4 Difference of 1 degree temperature was between
(A) 7 a.m. and 9 a.m.
(B) 9 a.m. and $11 \mathrm{a} . \mathrm{m}$.
(C) 9 a.m. and 1 p.m.
(D) 11 a.m. and 1 p.m.

Q. 5 The \% change in temperature from 5 a.m. to 7 a.m. is
(A) $10 \%$
(B) $20 \%$
(C) $30 \%$
(D) $4 \%$

Direction (Q. 6 to Q.9 : Read the following table and choose the correct answer from the given four alternative answers

| Class Intervals | $5-10$ | $10-15$ | $15-20$ | $20-25$ | $25-30$ | $30-35$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 18 | 27 | 19 | 14 | 3 | 8 |

Q. 6 The size of the class 25-30 is
(A) 5
(B) 25
(C) 30
(D) 27.5
Q. 7 The lower limit of the class $15-20$ is
(A) 20
(B) 15
(C) 5
(D) 35
Q. 8

The upper limit of the class $30-35$ is
(A) 30
(B) 32
(C) 35
(D) 65
Q. 9 The class mark of the class $5-10$ is
(A) 5
(B) 10
(C) 15
(D) 7.5

Direction (Q. 10 to 13) : The following graph shows the ages of teachers of a school :


Read the above graph (figure) and select the correct answer from the given four alternative answers :
Q. 10 The number of teachers who are 49 years and above is
(A) 6
(B) 8
(C) 4
(D) 10
Q. 11 The number of teachers in the age group 37-43 is
(A) 10
(B) 6
(C) 8
(D) 14
Q. 12 The number of teachers below 37 years is
(A) 10
(B) 24
(C) 14
(D) 18
Q. 13 Total number of teachers in the school is
(A) 42
(B) 32
(C) 40
(D) 38

Direction (Q. 14 to $\mathbf{Q} .17$ ) :The adjoining pie chart depicts the games played by 1500 students of a school. Each student plays only one game. Examine this graph and choose the correct answer.
Q. 14 Which game is liked least?
(A) Tennis
(B) Hockey
(C) Cricket
(D) Football
Q. 15 How many students play tennis?
(A) 250
(B) 500
(C) 425
(D) 200

Q. 16 What percentage of students play badminton?
(A) $33 \%$
(B) $13 \%$
(C) $33 \frac{1}{3} \%$
(D) $30 \%$
Q. 17 The ratio of the students who plays tennis to those who play cricket is
(A) $2: 1$
(B) $1: 2$
(C) $1: 1$
(D) $1: 3$
Q. 18 The number of times a particular entry occurs in a set of data is known as its :
(A) range
(B) class-size
(C) frequency
(D) class-interval
Q. 19 The difference between the highest and the lowest values of the observations in a given set of data is called as :
(A) range
(B) frequency
(C) class-size
(D) class-interval
Q. 20 The mid-value of a class-interval is called as :
(A) class-limit
(B) class-mark
(C) class-width
(D) range
Q. 21 The difference between upper and lower class limits of a group is called :
(A) class-limit
(B) class-size
(C) class-interval
(D) class-mark
Q. 22 Each group of a grouped frequency distribution is called :
(A) class-limit
(B) class-size
(C) class-interval
(D) class-mark

Directrion (Q. 23 to Q.27) : Study the graph, carefully and answer the questions. The graph given below, shows the

Q. 23 If the central government desires to give aid for speedy electrification, starting from states with least electrification which state will get fourth rank in the order of priority?
(A) C
(B) F
(C) D
(D) B
Q. 24 Which state has twice the percentage of villages not electrified in comparison to state A?
(A) B
(B) C
(C) D
(D) E
Q. 25 In case of state C , what per cent of villages are electrified?
(A) $65 \%$
(B) $25 \%$
(C) $20 \%$
(D) $80 \%$
Q. 26 How many states have atleast $60 \%$ or more electrified villages?
(A) 2
(B) 3
(C) 4
(D) 5
Q. 27 Which state has the maximum percentage of electrified villages ?
(A) A
(B) B
(C) C
(D) D

Direction (Q. 28 to Q .31 ) : The graph given below, shows the production of foodgrains of a country in different years. Questions 15 to 18 are based on this graph, study the graph and answer the questions

Q. 28 The sum of the production of foodgrains in the year 2002 and 2004 is equal to that in year
(A) 2003
(B) 2001
(C) 2000
(D) none of these
Q. 29 The different of the production of food grains for the year 2001 and 2005 is
(A) 50 tonnes
(B) 5 tonnes
(C) 5000 tonnes
(D) 500 tonnes
Q. 30 Minimum production during these years is
(A) 15 tonnes
(B) 150 tonnes
(C) 1500 tonnes
(D) 15000 tonnes
Q. 31 Maximum production during these years is in the year
(A) 2000
(B) 2001
(C) 2003
(D) 2005

Direction (Q. 32 to Q.35) : The following pie chart gives the marks obtained by a student in different subjects - English, Hindi, Mathematics, Science and Social Science is an examination.


Assuming that the total marks obtained for the examination are 540, answer the questions given below
Q. 32 The marks scored in Social Science are more than
(A) Mathematics
(B) Hindi
(C) English
(D) Science
Q. 33 The marks scored in Hindi and Mathematics exceeds the marks scored in English and Social Science by
(A) 30
(B) 50
(C) 70
(D) 60
Q. 34 The subject in which student scored 105 marks is
(A) English
(B) Hindi
(C) Science
(D) Social Science
Q. 35 The different of marks between English and Science is the same as between
(A) Science and Hindi
(B) Hindi and Social Science
(C) English and Hindi
(D) Mathematics and Social Science
Q. 36 A pictorial representation of the grouped data in which rectangles are formed with class intervals as the bases and the corresponding frequencies as the heights is known as
(A) bar graph
(B) pie chart
(C) histogram
(D) none of these
Q. 37 A circle divided into as many sectors as there are classes in a frequency distribution and size of sector proportionals to the frequency of the class is known as
(A) golden circle
(B) pie chart
(C) frequency circle
(D) super circle
Q. 38 The word experiment means
(A) two chemical reacts to give a new product
(B) two organic chemical react to give a new product
(C) an operation which can produce some well define outcomes
(D) none of these
Q. 39 When we perform an experiment then outcomes are known as equally liked if
(A) there is only one outcome
(B) all outcomes are same
(C) each outcome has the same chance of occurrence
(D) none of these

## ANSWER KEY

## CONCEPT APPLICATION LEVEL - II

| Q. 1 | C | Q. 2 | C | Q. 3 | B | Q. 4 | C | Q. 5 | D | Q. 6 | A | Q. 7 | B |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Q. 8 | C | Q. 9 | D | Q. 10 | A | Q. 11 | C | Q. 12 | B | Q. 13 | A | Q. 14 | A |
| Q. 15 | A | Q. 16 | C | Q. 17 | B | Q. 18 | C | Q. 19 | A | Q. 20 | B | Q. 21 | B |
| Q. 22 | C | Q. 23 | B | Q. 24 | D | Q. 25 | D | Q. 26 | B | Q. 27 | C | Q. 28 | B |
| Q. 29 | C | Q. 30 | D | Q. 31 | B | Q. 32 | C | Q. 33 | D | Q. 34 | B | Q. 35 | D |
| Q. 36 | C | Q. 37 | B | Q. 38 | C | Q. 39 | C |  |  |  |  |  |  |

