4 CUBES & CUBE ROOTS

THEORY

4.1 INTRODUCTION

Raising a Number to a Power

To raise a number to an integral power (second, third, fourth, etc.) we multiply the quantity by itself two, three four etc. times. The number repeated as a factor is called the base, the number which indicates how many times the base is to be used as a factor is called exponent of the power.

The second power is also called square & the third power is also called the cube of a number. The first power of a number is the number itself.

Similarly, if a is a rational number and n is a (positive integer) then

$a \times a \times a \times a............n \text{ times} = a^n$

4.2 CUBES

Cube of a number is that number raised to the power three. Thus if 'a' is the number then cube of 'a' is a^3

$a^3 = a \times a \times a$

The following table gives the cubes of first 10 natural numbers.

x	1	2	3	4	5	6	7	8	9	10
x^3	1	8	27	64	125	216	343	512	729	1000

For example, Cube of 3 is $3 \times 3 \times 3 = 3^3 = 27$

Cube of 0.1 is $0.1 \times 0.1 \times 0.1 = (0.1)^3 = 0.001$

4.2.1 Cubes of Natural Numbers

Cube of a natural number is that natural number raised to the power 3. Thus if 'm' is a natural number, then m^3 is the cube of m and we have $m^3 = m \times m \times m$

4.2.2 Perfect Cube

A natural number *n* is called a perfect cube if there exists a natural number a such that $n = a \times a \times a = a^3$

For example, 8 and 216 are perfect cubes because $8 = 2 \times 2 \times 2 = 2^3$, $216 = 6 \times 6 \times 6 = 6^3$

4.2.3 Cubes of Rational Numbers

Let $a = \frac{m}{n}$ be a rational number (m, n are non zero integers such that $n \neq \pm 1$) other than an integer, then the cube of a is defined as $a^3 = a \times a \times a$.

or $$\left(\frac{m}{n}\right)^3 = \frac{m}{n} \times \frac{m}{n} \times \frac{m}{n} = \frac{m^3}{n^3}$$

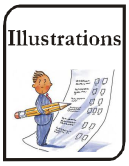

Illustration 1

Find cube of 2/3.

Solution

$$\left(\frac{2}{3}\right)^3 = \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \frac{8}{27}$$

4.2.4 Cubes of Negative Numbers

(i) If m is a positive integer, then

$(-m)^3 = -m \times -m \times -m = -m^3$

Thus for any positive integer m, $-m^3$ is the cube of $-m$.

For example, $(-2)^3 = -2 \times -2 \times -2 = -8$

Therefore, -8 is the cube of -2.

(ii) What is the number whose cube is -1331?

$$\begin{array}{r|l} 11 & 1331 \\ 11 & 121 \\ 11 & 11 \\ & 1 \end{array}$$

$1331 = 11 \times 11 \times 11$

but $-m^3$ is the cube of $-m$ for any positive integer m.

Therefore, 1331 is the cube of 11 and -1331 is the cube of -11.

4.2.5 Properties of Cubes of Numbers

1. Cube of an odd number is odd.

   $7^3 = 343$

   $13^3 = 2197$

   Thus, we note 7, 343, 13, 2197 are odd numbers.

2. Cube of an even number is even

   $8^3 = 512$

   $12^3 = 1728$

   Thus, we note 8, 512, 12, 1728 are even numbers.

3. Cube of a rational number $\frac{p}{q}$ is $\frac{p^3}{q^3}$

$$\left(\frac{2}{3}\right)^3 = \frac{2}{3}\times\frac{2}{3}\times\frac{2}{3} = \frac{2\times2\times2}{3\times3\times3} = \frac{8}{27}$$

$$\left(\frac{-8}{9}\right)^3 = \frac{-8}{9}\times\frac{-8}{9}\times\frac{-8}{9} = \frac{(-8)\times(-8)\times(-8)}{9\times9\times9} = \frac{-512}{729}$$

4. If a number is a multiple of 3 then the cube of that number is a multiple of 27

   $6^3 = 6 \times 6 \times 6 = 216$ $(6 = 2 \times 3,\ 6^3 = 216 = 8 \times 27)$

   $12^3 = 12 \times 12 \times 12 = 1728$ $(12 = 4 \times 3,\ 12^3 = 1728 = 64 \times 27)$

Properties of Cubes of Numbers

(1) Cubes of all odd natural numbers are odd

(2) Cubes of all even natural numbers are even.

(3) Cube of a natural number which is multiple of 3 is a multiple of 27.

(4) Cube of a number which ends in a zero, ends in three zeros.

(5) The cube of a negative integer is a negative.

(6) Cube of a rational number is the cube of the numerator divided by the cube of the denominator.

4.3 CUBE ROOTS

A natural number m is the cube root of a natural number n if $m^3 = n$.

The cube root of a number 'n' is denoted by the symbol $\sqrt[3]{n}$

Illustration 2

The cube root of 8 is 2. In symbols it is expressed as $\sqrt[3]{8} = 2$.

Solution

The symbol $\sqrt[3]{\ }$ for the cube root is very much similar to the symbol for square root. The only difference is that whereas in the case of square root we use the symbol '$\sqrt{\ }$' for the cube root we use the same symbol '$\sqrt{\ }$' but with a '3' which indicates that we are taking a cube root.

4.3.1 Cube Root of a perfect cube by factors

Step–1:

(i) Express the given number as a product of prime factors.

(ii) Make triplets of similar factors

(iii) The product of the prime factors after taking one factor out of every triple will give the cube root of the number.

Illustration 3

Find the cube root of 3375 by using prime factorization method.

Solution

$3375 = 5 \times 5 \times 5 \times 3 \times 3 \times 3$

$$\sqrt[3]{3375} = \sqrt[3]{5 \times 5 \times 5 \times 3 \times 3 \times 3}$$

$$= \sqrt[3]{(5 \times 3)(5 \times 3)(5 \times 3)} = 15$$

5	3375
5	675
5	135
3	27
3	9
3	3
	1

4.3.2 Cube root of a negative integeral perfect cube

We know that for any positive integer x, $-x$ is a negative integer such that

$$(-x)^3 = (-x) \times (-x)\ (-x) = -x^3$$

$$\therefore \quad \sqrt[3]{-x^3} = -x$$

$$\Rightarrow \quad \sqrt[3]{-x^3} = -\sqrt[3]{x^3}$$

Thus, the cube root of a negative perfect cube is a negative of the cube root of its absolute value.

In otherwords, to find the cube root of a negative perfect cube, we find the cube root of its absolute value and multiply it by -1.

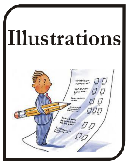

Illustration 4

Find the cube root of –343.

Solution

After finding cube root of 343, we put minus sign before it

7	343
7	49
7	7
	1

$$\sqrt[3]{-343} = -7$$

4.3.3 Cube Root of a Product of Integers

In order to find the cube root of the product of two integers, a and b we have

$$\sqrt[3]{ab} = \sqrt[3]{a} \times \sqrt[3]{b}$$

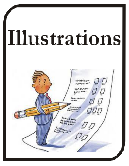

Illustration 5

Find the cube root of –216 × 1728.

Solution

$$\sqrt[3]{-216 \times 1728} = \sqrt[3]{-216} \times \sqrt[3]{1728}$$

$$= -\sqrt[3]{216} = -6 \times 12 = -72$$

4.3.4 Cube Root of a Rational Number

For any rational number a/b we have

$$\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$$

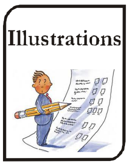

Illustration 6

Find the cube root of $\frac{1331}{4096}$

Solution

$$\sqrt[3]{\frac{1331}{4096}} = \frac{\sqrt[3]{1331}}{\sqrt[3]{4096}} = \frac{11}{16}$$

4.3.5 Cube Root of Perfect Numbers

(A) Using ones and tens Method

This method can be used to find cube roots of perfect cubes having at most 6 digits. CUBES of number ending in 0, 1, 4, 5, 6 and 9 also ends in 0, 1, 4, 5, 6 and 9.

However, the cube of a number ending in 2 ends in 8 and vice-versa. Similarly, the cube of a number ending in 3 ends in 7 and vice-versa.

Thus, by looking at the one's digit of a perfect cube number, we can determine the one's digit of its cube roots.

Let us consider the following steps to find the two digits of the cube root of PERFECT cubes.

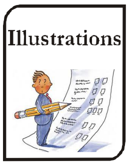

Illustration 7

Find the cube root of the following :

(a) 2197 **(b) 531441** **(c) 91125**

Solution

(a) $\sqrt[3]{2197}$ → The number is ending with 7.

T O

Step-1: ? 3 (Numbers ending in 7 will have their cube root ending is 3 as $3^3 = 27$)

Step 2 : 2 ~~1 9 7~~ (Strike out last three i.e, 'O', 'T', 'H' digits of the number from the right)

T O

Step 3 : 1 3 (For ten's digit, think of a number whose cube is smaller than 2 –

Since $1^3 < 2$ the number left after striking the last three digits)

and $2^3 > 2$ Hence, number is ten's place will be 1.

Hence $\sqrt[3]{2197} = 13$

(b) $\sqrt[3]{531441}$

Since T O Numbers ending in 1 will have their cube root ending in 1 as $1^3 = 1$

$8^3 < 531 \rightarrow$ [8] 1 After striking the last three digts, Number left is 531.

and $9^3 > 531$

Hence, $\sqrt[3]{531441} = 81$

(c) $\sqrt[3]{91125} =$ T O

For ten's digit $\leftarrow 4$ $5 \rightarrow$ For one's digit

($4^3 < 91$ and $5^3 > 91$) (Numbers ending in 5 have their cube roots ending in 5 as $5^3 = 125$)

Hence, $\sqrt[3]{91125} = 45$

(B) By Prime Factorisation Method :

Procedure for finding the cube root of perfect cubes

Step : 1 Write the prime factors of the number.

Step : 2 Make triplets (groups of three) of equal factors obtained in step 1.

Step : 3 Write one factor responding to each triplet obtained in step 2.

Step : 4 Find the product of the factors obtained in step 3.

The product obtained in step 4 will be the required cube root.

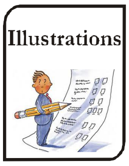

Illustration 8

Find the cube roots of the numbers :

(a) 4096 (b) 9261 (c) –6859

Solution

(a) 4096

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Thus, $4096 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$\sqrt[3]{4096} = \sqrt[3]{(2\times2\times2)\times(2\times2\times2)\times(2\times2\times2)\times2\times2\times2}$

$\sqrt[3]{4096} = 2 \times 2 \times 2 \times 2 = 16$

This process can be exhibited as follows also :

$$4096 = \underbrace{(2 \times 2 \times 2)}_{2^3} \times \underbrace{(2 \times 2 \times 2)}_{2^3} \times \underbrace{(2 \times 2 \times 2)}_{2^3} \times \underbrace{(2 \times 2 \times 2)}_{2^3}$$

$\sqrt[3]{4096} = 2 \times 2 \times 2 \times 2 = 16$

(b) 9261

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$9261 = 3 \times 3 \times 3 \times 7 \times 7 \times 7$

$\sqrt[3]{9261} = \sqrt[3]{(3\times3\times3)\times(7\times7\times7)}$

$= 3 \times 7 = 21$

(c) –6859

19	6859
19	361
19	19
	1

(The cube root of negative number is negative)

$-6859 = (-19) \times (-19) \times (-19) = (-19)^3$

$\sqrt[3]{-685} = -19$

Or

$-6859 = (-1) \times 19 \times 19 \times 19$

$= (-1)^3 \times (19)^3$

$\sqrt[3]{-6859} = -1 \times 19 = -19$

4.3.6 Cube Roots of Decimal Numbers

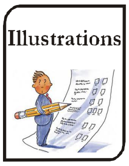

Illustration 9

Evaluate $\sqrt[3]{0.125}$

Solution

$$\sqrt[3]{0.125} = \sqrt[3]{\frac{125}{1000}} = \frac{\sqrt[3]{125}}{\sqrt[3]{1000}} = \frac{5}{10} = 0.5$$

Illustration 10

Evaluate $\sqrt[3]{0.002197}$

Solution

$$\sqrt[3]{0.002197} = \sqrt[3]{\frac{2197}{1000000}} = \frac{\sqrt[3]{2197}}{\sqrt[3]{1000000}} = \frac{13}{100} = 0.13$$

Illustration 11

Evaluate $\sqrt[3]{0.027 \times 2.744}$

Solution

$$\sqrt[3]{0.027 \times 2.744} = \sqrt[3]{0.027} \times \sqrt[3]{2.744}$$

$$= \sqrt[3]{\frac{27}{1000}} \times \sqrt[3]{\frac{2744}{1000}} = \frac{3}{10} \times \frac{14}{10} = 0.3 \times 1.4 = 0.42$$

Illustration 12

Find the smallest number by which 16875 be divided to get a perfect cube. Find the cube root of the number thus obtained.

Solution

3	16875
3	5625
3	1875
5	625
5	125
5	25
5	5
	1

Thus, $16875 = 3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5$

Since by making triplets of equal factors, 5 is left ungrouped.

$\therefore$ On dividing 16875 by 5, we get 3375 which is a perfect cube.

$\sqrt[3]{3375} = 3 \times 5 = 15$

4.3.7 Method of successive subtraction to find cube root

We subtract the numbers, 1, 7, 19, 37, 61, 91, 127, 169, ... successively till we get zero. The number of subtractions will give the cube root of the number. The number 1, 7, 19, 37, 61, 91, 127, 169, ... are obtained by putting n = 1, 2, 3, ... in $1 + n \times (n - 1) \times 3$

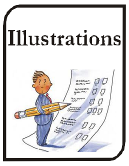

Illustration 13

Find the cube root of 343 using the method of successive subtraction.

Solution

343 – 1 = 342, 342–7 = 335, 335 – 19 = 316, 316 – 37 = 279,

279 – 61 = 218, 218 – 91 = 127, 127 – 127 = 0

∴ The number of subtractions to yield zero is 7.

∴ $\sqrt[3]{343} = 7$

Illustration 14

Examine if 130 is a perfect cube. If not, then find the smallest number that must be subtracted form 130 to obtain a perfect cube.

Solution

130 – 1 = 129, 129 – 7 = 122, 122 – 19 = 103, 103 – 37 = 66, 66 – 61 = 5.

The next number to be subtracted is 91, which is greater than 5. Therefore, the process of successive subtraction does not give zero. Hence, 130 is not a perfect cube. If we subtract 5 from 130, the above process will give zero after 5 successive subtractions.

4.4 METHOD TO CHECK WHETHER A GIVEN NUMBER IS A PERFECT CUBE OR NOT BY PRIME FACTORISATION

1. Find the prime factors of the given number.
2. Group the prime factors as triplets (group of 3) of equal factors.
3. If all the triplets are complete i.e., all the factors have been grouped and each group having 3 identical factors each, then the number is a perfect cube. If any triplet is left ungrouped then the given number is not a perfect cube.

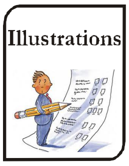

Illustration 15

Check whether following numbers are perfect cubes or not :

(a) 4096 **(b) 6859** **(c) 52728** **(d) 88434**

Solution

(a)

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$4096 = (2 \times 2 \times 2) \times (2 \times 2 \times 2)\ (2 \times 2 \times 2) \times (2 \times 2 \times 2)$

As all the triplets of 2 are complete, 4096 is a perfect cube,

(b)

19	6589
17	361
19	19
	1

$6859 = (19 \times 19 \times 19)$

As the triplet of 19 is complete, 6859 is a perfect cube.

(c)

2	52728
2	26364
2	13182
3	6591
13	2197
13	169
13	13
	1

$52728 = (2 \times 2 \times 2) \times 3 \times (13 \times 13 \times 13)$

As the triplets of 3 is incomplete, 52728 is not perfect cube.

(d)

2	88434
3	44217
3	14739
17	4913
17	289
17	17
	1

$88434 = 2 \times (3 \times 3) \times (17 \times 17 \times 17)$

As the triplets of $2 \times (3 \times 3)$ are incomplete, so 88434 is not a perfect cube.

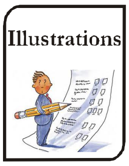

Illustration 16

Check whether 27648 is a perfect cube or not. If not, find the least number by which 27648 must be multiplied so that the product is a perfect cube. Also write the perfect cube so obtained.

Solution

27648

2	27648
2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$27648 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times 2 \times (3 \times 3 \times 3)$

As the triplet of 2 is incomplete, so 27648 is not perfect cubes. We multiply 27648 by $2 \times 2 = 4$ to obtain a perfect cube.

Note : the square root of a negative number does not exist whereas cube root of a negative number exists and is a negative number.

[For example, $\sqrt{-64}$ does not exist since there is no number whose square is – 64 [$\because$ $8^2 = 64$, $(-8)^2 = 64$] but $\sqrt[3]{-64} = -4$ since $(-4)^3 = (-4)(-4)(-4) = -64$]

SOLVED EXAMPLES

Q.1 Find the cube root of

(a) 474552 (b) $\frac{729}{2197}$ (c) 216×1000 (d) -9261×512

(e) -10648

Sol. (a) $474552 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 13 \times 13 \times 13$

2	474552
2	237276
2	118638
3	59319
3	19773
3	6591
13	2197
13	169
13	13
	1

Cube root of 474552 is $2 \times 3 \times 13$ i.e. 78.

(b) $\frac{729}{2197} = \frac{9\times9\times9}{13\times13\times13}$

cube root of $\frac{729}{2197}$ is $\frac{9}{13}$

(c) $216 \times 1000 = 6 \times 6 \times 6 \times 10 \times 10 \times 10$

cube root of 216×1000 is 6×10 i.e. 60

(d) $-9261 \times 512 = -3 \times 3 \times 3 \times 7 \times 7 \times 7 \times 2^9$

$= -3^3 \times 7^3 \times 2^9$

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$\therefore$ cube root of -9216×512 is

$-3 \times 7 \times 2^3 = -21 \times 8 = -168$

(e) $-10648 = -2^3 \times 11^3$

Hence, cube root of -10648 is -2×11 i.e. -22

2	10648
2	5324
2	2662
11	1331
11	121
11	11
	1

Q.2 By what smallest natural number should –250 be divided so that quotient becomes a perfect cube?

Sol. $-250 = -2 \times 5 \times 5 \times 5 = -2^1 \times 5^3$

–250 must be divided by 2.

Q.3 By what smallest natural number should –6125 be multiplied so that the product becomes a perfect cube.

Sol. $-6125 = -5^3 \times 7^2$

–6125 should be multiplied by 7

5	6125
5	1225
5	245
7	49
7	7
	1

Q.4 If the volume of a cube is 512 m^3. What is the length of one side of cube.

Sol. Volume of Cube = a^3

$\Rightarrow \quad 512\ m^3 = a^3 \quad \Rightarrow \quad a^3 = 2^9\ m^3 \quad \Rightarrow a = \sqrt[3]{2^9 m^3} = 2^3\ m = 8\ m$

Q.5 Three numbers are in the ratio 1 : 2 : 3. The sum of their cubes is 98784. Find the numbers.

Sol. Let the numbers be x, 2x and 3x. ATQ,

$x^3 + (2x)^3 + (3x)^3 = 98784$

$\Rightarrow \quad x^3 + 8x^3 + 27x^3 = 98784 \quad \Rightarrow \quad 36\ x^3 = 98784$

$\Rightarrow \quad x^3 = \frac{98784}{36} = \frac{2^5 \times 3^2 \times 7^3}{2^2 \times 3^2} \quad \Rightarrow \quad x^3 = 2^{5-2} \times 3^{2-2} \times 7^3$

$\Rightarrow \quad x^3 = 2^3 \times 7^3 \quad \Rightarrow \quad x = 2 \times 7 = 14$

Hence, The numbers are $\begin{Bmatrix} x = 14 \\ 2x = 28 \\ 3x = 42 \end{Bmatrix}$.

2	98784
2	49392
2	24696
2	12348
2	6174
3	3087
7	1029
7	147
7	21
3	3
	1

Q.6 Find the volume of a cube where surface area is 384 m^2.

Sol. Let the side of cube is a.

$\therefore \quad 6\ a^2 = 384\ m^2 \quad \Rightarrow \quad a^2 = \frac{384}{6} m^2 = 64 m^2 \quad \Rightarrow \quad \boxed{a = 8m}$

Volume of cube = $a^3 = 8^3 = 512$

Q.7 Identify the perfect cubes.
8, 16, 36, 49, 64, 100

Sol. $8 = (2)^3$, $64 = (4)^3$

Q.8 Find the cubes of the following :

(a) 0.06 (b) $3 + \frac{1}{5}$ (c) $4\frac{2}{3}$ (d) $5 - 2\frac{1}{3}$

Sol. (a) $(0.06)^3 = 0.06 \times 0.06 \times 0.06 = 0.000216$

(b) $\left(3+\frac{1}{5}\right)^3 = \left(\frac{8}{5}\right)^3 = \frac{8^3}{5^3} = \frac{512}{125}$

(c) $\left(4\frac{2}{3}\right)^3 = \left(\frac{14}{3}\right)^3 = \frac{14^3}{3^3} = \frac{2744}{27}$

(d) $\left(5-2\frac{1}{3}\right)^3 = \left(5-\frac{7}{3}\right)^3 = \left(\frac{8}{3}\right)^3 = \frac{8^3}{3^3} = \frac{512}{27}$

Q.9 Find the cube roots of the following
(a) 4096 (b) 35937 (c) 474552 (d) 729000

Sol. (a) $\sqrt[3]{4096} = \sqrt[3]{2^{12}} = -2$

(b) $\sqrt[3]{35937} = \sqrt[3]{3^3 \times 11^3} = 3 \times 11 = 33$

3	35937
3	11979
3	3993
11	1331
11	121
11	11
	1

(c) $\sqrt[3]{474552} = \sqrt[3]{2^3 \times 3^3 \times 13^3} = 2 \times 3 \times 13 = 6 \times 13 = 78$

2	474552
2	237276
2	118638
3	59319
3	19773
3	6591
13	2197
13	13
	1

(d) $\sqrt[3]{729000} = \sqrt[3]{9^3 \times 10^3} = 9 \times 10 = 90$

Q.10 By what smallest natural number should –250 be divided so that the quotient becomes a perfect cube?

Sol. $-250 = -2 \times 5 \times 5 \times 5$ it must be divided by 2

Q.11 Evaluate :

(a) $\sqrt[3]{8000}$ (b) $\sqrt[3]{\frac{-0.027}{8}}$

(c) $\sqrt[3]{2744} - \sqrt{169}$ (d) Find $\sqrt[3]{74088} : \sqrt[3]{9261}$

Sol. (a) $\sqrt[3]{8000} = \sqrt[3]{(20)^3} = 20$

(b) $\sqrt[3]{\frac{-0.027}{8}} = \sqrt[3]{-\left(\frac{-0.3}{2}\right)^3} \; \sqrt[3]{\left(\frac{-0.3}{2}\right)^3} = \frac{-0.3}{2} = \frac{-3}{20}$

(c) $\sqrt[3]{2744} - \sqrt{169} = \sqrt[3]{(14)^3} - \sqrt{(13)^2} = 14 - 13 = 1$

(d) $\frac{\sqrt[3]{74088}}{\sqrt[3]{9261}} = \sqrt[3]{\frac{74088}{9261}} = \sqrt[3]{\frac{2^3 \times 3^3 \times 7^3}{3^3 \times 7^3}} = \sqrt[3]{2^3} = 2$

2	74088
2	37044
2	18522
3	9261
3	3037
3	1029
7	343
7	49
7	7
	1

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

Q.12 Find the cube root of the following by the method of successive subtraction of the numbers 1, 7, 19, 37, 61, 91, 127, 169, 217, 271, 331, 397, ...

(a) 125 [Ans. 5]

(b) 343 [Ans. 7]

(c) 2197 [Ans. 13]

CONCEPT APPLICATION LEVEL - I [NCERT Questions]

EXERCISE 1

Q.1 Which of the following numbers are not perfect cubes ?

(i) 216 (ii) 128 (iii) 1000 (iv) 100 (v) 46656

Sol. (i) 216

2	216
2	108
2	54
3	27
3	9
3	3
	1

By prime factorisation,

$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$ (grouping the factors in triplets)

$= 2^3 \times 3^3$ (by laws of exponents)

$= (2 \times 3)^3$ (by laws of exponents)

$= 6^3$, which is a perfect cube.

Therefore, 216 is a perfect cube

(ii) 128

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

By prime facorisation,

$128 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 2$ (grouping the factors in triplets)

$= 2^3 \times 2^3 \times 2$

In the above factorisation, 2 remains after grouping the 2's in triplets.

Therefore, 128 is not a perfect cube.

(iii) 1000

2	1000
2	500
2	250
5	125
5	25
5	5
	1

By prime factorisation,

$1000 = \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5}$ (grouping the factors in triplets)

$= 2^3 \times 5^3$ (by laws of exponents)

$= (2 \times 5)^3 = 10^3$, (by law of exponents)

which is a perfect cube. Therefore, 1000 is a perfect cube.

(iv) **100**

2	100
2	50
5	25
5	5
	1

By prime factorisation

$100 = 2 \times 2 \times 5 \times 5$

In the above factorisation, 2×2, 5×5 remains when try to group the factors in triplets.

Therefore, 100 is not a perfect cube.

(v) **46656**

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
2	729
3	243
3	81
3	27
3	9
3	3
	1

By prime factorisation.

$46656 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$ (grouping the factors in triplets)

$= 2^3 \times 2^3 \times 3^3 \times 3^3$

$= 36^3$, which is a perfect cube.

Q.2 Find the smallest number by which each of the following numbers must be multiplied to obtain a perfect cube.

(i) 243 (ii) 256 (iii) 72 (iv) 675 (v) 100

Sol. (i) 243

3	243
3	81
3	27
3	9
3	3
	1

By prime factorisation,

$243 = \underline{3 \times 3 \times 3} \times 3 \times 3$ (grouping the factors in triplets)

The prime factor 3 does not appear in a group of three.

Therefore, 243 is not a perfect cube. To make it a cube, we need one more 3. In that case

$243 \times 3 = \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} = 729$, which is a perfect cube.

Hence, the smallest number by which 243 should be multiplied to make a perfect cube is 3.

(ii) 256

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

By prime factorisation,

$256 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 2 \times 2$ (grouping the factors in triplets)

In the above factorisation 2 remains after grouping 2's in triplets. Therefore, 128 is not a perfect cube. To make it a perfect cube, we need one 2's more. In that case,

$256 \times 2 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

$= 2^3 \times 2^3 \times 2^3$ (by laws of exponents)

$= 8^3 = 512$,

which is a perfect cube.

Hence, the smallest number by which 256 must be multiplied to obtain a perfect cube is 2.

The resulting perfect cube is 512 (= 8^3).

(iii) 72

2	72
2	36
2	18
3	9
3	3
	1

By prime factorisation,

$72 = \underline{2 \times 2 \times 2} \times 3 \times 3$ (grouping the factors in triplets)

The prime factors 3 does not appear in a group of three. Therefore, 72 is not a perfect cube. To make it a cube, we need one more 3. In that case,

$72 \times 3 = \underline{2 \times 2 \times 2} \times 3 \times 3$ (grouping the factors in triplets)

$= 2^3 \times 3^3$ (by laws of exponents)

$= (2 \times 3)^3$ (by laws of exponents)

$= 6^3$, which is a perfect cube.

Hence, the smallest number by which 72 must be multiplied to obtain a perfect cube is 3.

(iv) **675**

3	675
3	225
3	75
5	25
5	5
	1

By prime factorisation,

$675 = 3 \times 3 \times 3 \times 5 \times 5$ (grouping the factors in triplets)

The prime factor 5 does not appear in a group of three. Therefore, 675 is not a perfect cube. To make it a cube, we need one more 5. In that case,

$675 \times 5 = \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5} = 3^3 \times 5^3$ (by laws of exponents)

$= (3 \times 5)^3$ (by laws of exponents)

$= 15^3$, which is a perfect cube.

Hence, the smallest number by which 675 must be multiplied to obtain a perfect cube is 5.

The resulting perfect cube is 3375 (= 15^3).

(v) **100**

2	100
2	50
5	25
5	5
	1

By prime factorisation,

$100 = 2 \times 2 \times 5 \times 5$ (grouping the factors in triplets)

The prime factors 2 and 5 do not appear in a group of three. Therefore, 100 is not a perfect cube. To make it a perfect cube, we need one 2 and one 5 more. In that case,

$100 \times 2 \times 5 = \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5} = 2^3 \times 5^3$ (by laws of exponents)

$= 10^3$,

which is a perfect cube.

Hence, the smallest number by which 100 must be multiplied to obtain a perfect cube is $2 \times 5 = 10$.

The resulting perfect cube is 1000 (= 10^3).

Q.3 Find the smallest number by which each of the following numbers must be divided to obtain a perfect cube.

(i) **81** **(ii)** **128** **(iii)** **135** **(iv)** **192**

(v) **704**

Sol. **(i)** **81**

3	81
3	27
3	9
3	3
	1

By prime factorisation,

$81 = 3 \times 3 \times 3 \times 3$ (grouping the factors in triplets)

In the above factorisation 3 remains after grouping the 3's in triplets. Therefore, 81 is not a perfect cube. If we divided the number by 3, then in the prime factorisation of the quotient, this 3 will not remain. In that case,

$81 \div 3 = \underline{3 \times 3 \times 3}$

$= 3^3$, which is a perfect cube

Hence, the small hole number by which 81 must be divided to obtain a perfect cube is 3.

The resulting perfect cube is $27 (= 3^3)$

(ii) **128**

2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

By prime factorisation,

$128 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 2$ (grouping the factors in triplets)

In the above factorisation, 2 remains after grouping the 2's in triplets. Therefore, 128 is not a perfect cube. If we divide the number by 2, then in the prime factorisation of the quotient, this 2 will not remains. In that case,

$128 \div 2 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

$= 2^3 \times 2^3$ (by laws of exponents)

$= (2 \times 2)^3$ (by laws of exponents)

$= 4^3$, which is a perfect cube.

Hence ,the smallest whole number by which 128 must be divided to obtain a perfect cube is 2.

The resulting perfect cube is $64 (= 4^3)$

(iii) **135**

3	135
3	45
3	15
5	5
	1

By prime factorisation,

$135 = 3 \times 3 \times 3 \times 5$ (grouping the factors in triplets)

The prime factor 5 does not appear in a group of three. So 135 is not a perfect cube. In the factorisation 5 appears only ones. If we divide 135 by 5, then the prime factorisation of the quotient will not contain 5. In that case,

$135 \div 5 = \underline{3 \times 3 \times 3} = 3^3$, which is a perfect cube. (by laws of exponents)

Hence, the smallest whole number by which 135 must be divided to obtain a perfect cube is 5.

The resulting perfect cube is $27(=3^3)$

(iv) 192

2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

By prime factorisation,

$192 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 3$ (grouping the factors in triplets)

The prime factor 3 does not appear in a group of three. So 192, 3 appears only once. So if we divide the number by 3, then the prime factorisation of the quiotient will not contain 3. In that case,

$192 \div 3 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

$= 2^3 \times 2^3$ (by laws of exponents)

$= (2 \times 2)^3$ (by laws of exponents)

$= 4^3$, which is a perfect cube.

Hence, the smallest whole number by which 192 must be divided to obtain a perfect cube is 3. The resulting perfect cube is $64(= 4^3)$

(v) 704

2	704
2	352
2	176
2	88
2	44
2	22
11	11
	1

By prime factorisation,

$704 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 11$ (grouping the factors in triplets)

The prime factor 11 does not appear in a group of three. So, 704 is not a perfect cube.

In the factorisation 11 appears only one time. So if we divide 704 by 11, then the prime factorisation of the quotient will not contain 11. In that case,

$704 \div 11 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

$= 2^3 \times 2^3$ (by laws of exponents)

$= (2 \times 2)^3$

by laws of exponents

$= 4^3$, which is a perfect cube.

Hence, the smallest whole number by which 704 must be divided to obtain a perfect cube is 11. The resulting perfect cube is $64 (= 4^3)$

Q.4 Parikshit makes a cuboid of plasticine of sides 5 cm, 2 cm, 5 cm. How may such cuboids will he need to form a cube ?

Sol. Volume of a cuboid = $5 \times 2 \times 5$ cm^3.

Since there is only one 2 and only two 5 in the prime factorisation, so, we need $2 \times 2 \times 5$, i.e., 20 to make a perfect cube. Therefore, we need 20 such cuboids to make a cube.

EXERCISE 2

Q.1 Find the cube root of each of the following numbers by prime factorisation method :

(i) 64 (ii) 512 (iii) 10648 (iv) 27000
(v) 15625 (vi) 13824 (vii) 110592 (viii) 46656
(ix) 175616 (x) 91125

Sol. (i) 64

2	64
2	32
2	16
2	8
2	4
2	2
	1

Prime factorisation of 64 is

$\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$ (grouping the factors in tripletes)

$= 2^3 \times 2^3 = (2 \times 2)^3 = 4^3$

By laws of exponents. Therefore, $\sqrt[3]{64} = 4$

(ii) 512

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

$\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$ (grouping the factors in triplets)

$= 2^3 \times 2^3 \times 2^3 = (2 \times 2 \times 2)^3 = 8^3$ (by laws of exponents)

Therefore, $\sqrt[3]{512} = 8$

(iii) **10648**

2	10648
2	5324
2	2662
11	1331
11	21
11	11
	1

Prime factorisation of 10648 is

$2 \times 2 \times 2 \times 11 \times 11 \times 11$ (grouping the factors in triplets)

$= 2^2 \times 11^3$ by laws of exponents

Therefore, $\sqrt[3]{10648} = 2 \times 11 = 22$

(iv) **27000**

2	27000
2	13500
2	6750
3	3375
3	1125
3	375
5	125
5	25
5	5
	1

Prime factorisation of 27000 is

$2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5$ (grouping the factors in triplets)

$= 2^3 \times 3^3 \times 5^3$ (by laws of exponents)

Therefore, $\sqrt[3]{2700} = 2 \times 3 \times 5 = 30$

(v) **15625**

2	15625
5	3125
5	625
5	125
5	25
5	5
	1

Prime factorisation of 15625 is

$5 \times 5 \times 5 \times 5 \times 5 \times 5$ (grouping the factors in triplets)

$= 5^3 \times 5^3 = (5 \times 5)^3 = 25^3$ (by laws of exponents)

Therefore, $\sqrt[3]{15625} = 5 \times 5 = 25$

(vi) **13824**

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

Prime factorisation of 13824 is

$\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$ (grouping the factors in triplets)

$= 2^3 \times 2^3 \times 2^3 \times 3^3$

$= (2 \times 2 \times 2 \times 3)^3 \times 24^3$ (by laws of exponents)

Therefore, $\sqrt[3]{13824} = 2 \times 2 \times 2 \times 3 = 24$

(vii) **110592**

2	110592
2	55296
2	27648
2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

Prime factorisation of 110592 is

$\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

(grouping the factors in triplets)

$= 2^3 \times 2^3 \times 2^3 \times 2^3 \times 3^3$

$= (2 \times 2 \times 2 \times 3)^3 = 24^3$ (by laws of exponents)

Therefore, $\sqrt[3]{110592} = 2 \times 2 \times 2 \times 2 \times 3 = 48$

(viii) **46656**

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

Prime factorisation of 46656 is

$\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}$ (grouping the factors in triplets)

$= 2^3 \times 2^3 \times 3^3 \times 3^3$

$= (2 \times 2 \times 3 \times 3)^3 = 36^3$ (by laws of exponents)

Therefore, $\sqrt[3]{46656} = 2 \times 2 \times 3 \times 3 = 36$

(ix) **175616**

2	175616
2	87808
2	43904
2	21952
2	10976
2	5488
2	2744
2	1372
7	686
7	343
7	49
7	7
	1

Prime factorisation of 175616 is $\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{7 \times 7 \times 7}$

(Grouping the factors in triplets)

$= 2^3 \times 2^3 \times 2^3 \times 7^3$

$= (2 \times 2 \times 2 \times 7)^3 = 56^3$ (by laws of exponents)

Therefore, $\sqrt[3]{175616} = 2 \times 2 \times 2 \times 7 = 56$

(x) **91125**

3	91125
3	30375
3	10125
3	3375
3	1125
5	375
5	125
5	25
5	5
	1

Prime factorisation of 91125

$\underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$ (grouping the factors in triplets)

$= 3^3 \times 3^3 \times 5^3$

$= (3 \times 3 \times 5)^3 = 45^3$ (by laws of exponents)

Therefore, $\sqrt[3]{91125} = 3 \times 3 \times 5 = 45$

Q.2 State true of false :

(i) Cube of any odd number is even.

(ii) A perfect cube does not end with two zeros.

(iii) If square of a number ends with 5, then its cube ends with 25

(iv) There is no perfect cube which ends with 8.

(v) The cube of a two digit number may be a three digit number.

(vi) The cube of a two digit number may have seven or more digits.

(vii) The cube of a single digit number may be a single digit number.

Sol. (i) False

(ii) True

(iii) False ($15^2 = 225$, $15^3 = 3375$)

(iv) False ($12^3 = 1728$)

(v) False ($10^3 = 1000$, $99^3 = 970299$)

(vi) False ($10^3 = 1000$, $99^3 = 970299$)

(vii) True ($1^3 = 1$; $2^3 = 8$)

Q.3 You are told that 1,331 is a perfect cube. Can you guess without factorisation what is its cube root ? Similarly, guess the cube roots of 4913, 12167, 32768.

Sol. By guess, Cube root of 1331 = 11

Similarly, Cube root of 4913 = 17

Cube root of 12167 = 23

Cube root of 32768 = 32

EXPLANATIONS

(i) Cube root of 1331 : The given number is 1331

Step 1. Form groups of three starting from the rightmost digit of 1331.1 331

In this case, one group i.e., 331 has three digits whereas 1 has only 1 digit.

Step2. Take 331

The digit 1 is at one's place We take the one's place of the required cube root as 1.

Step3. Take the other group, i.e., 1

Cube of 1 is 1.

Take 1 as ten's place of the cube root of 1331.

Thus, $\sqrt[3]{1331} = 11$

(ii) Cube root of 4913 : The given number is 4913

Step 1. From groups of three starting from the rightmost digit of 4913.

In this case one group,. i.e., 913 has three digits whereas 4 has only one digit.

Step 2. Take 931.

The digit 3 is at one's place. We take the one's place of the required cube root as 7.

Step 3. Take the other group, i.e., 4.

Cube of 1 is 1 and cube of 2 is 8. 4 lies between 1 and 8. The smaller number among 1 and 2 is 1. The one's place of 1 is 1 itself. Take 1 as ten's place of the cube root of 4913.

Thus, $\sqrt[3]{4913} = 17$.

(iii) Cube root of 12167 : The given number is 12167.

Step 1. Form groups of three starting from the rightmost digit of 12167.

12167. In this case, one group, i.e., 167 has three digits whereas 12 has only two digits.

Step 2. Take 167

The digit 7 is at its one's place. We take the one's place of the required cube root as 3.

Step 3. Take the other group, i.e., 12. Cube of 2 is 8 and cube of 3 is 27.12 lies between 8 and 27. The smaller among 2 and 3 is 2.

The one's place of 2 is 2 itself. Take 2 as ten's place of the cube root of 12167.

Thus, $\sqrt[3]{12167} = 23$

(iv) Cube root of 32768 :

Step 1. From groups of three starting from the rightmost digit of 32768.

32 768. In this case one group, i.e., 768 has three digits whereas 32 has only two digits.

Step2. Take 768.

The digit 8 is at its one's place.

We take the one's place of the required cube root as 2.

Step 3. Take the other group, i.e., 32. Cube of 3 is 27 and cube of 4 is 64.

The smaller number between 3 and 4 is 3.

The one's place of 3 is 3 itself.

Take 3 as ten's place of the cube root of 32768.

Thus, $\sqrt[3]{32768} = 32$

TRY THESE

Q.1 Find the one's digit of the cube of each of the following numbers.

(i) 3331	**(ii) 8888**	**(iii) 149**	**(iv) 1005**
(v) 1024	**(vi) 77**	**(vii) 5022**	**(viii) 53**

Sol. (i) 3331

$\because$ Unit digit of the number = 1

$\therefore$ Unit digit of the cube of the number = 1

(ii) 8888

$\because$ Unit digit of the number = 8

$\therefore$ Unit digit of the cube of the number = 2.

(iii) 149

$\because$ Unit digit of the number = 9

$\therefore$ Unit digit of the cube of the number = 9

(iv) 1005

$\because$ Unit digit of the number = 5

$\therefore$ Unit digit of the cube of the number = 5

(v) 1024

$\because$ Unit digit of the number = 4

$\therefore$ Unit digit of the cube of the number = 4

(vi) 77

$\because$ Unit digit of the number = 7

$\therefore$ Unit digit of the cube of the number = 3

(vii) 5022

$\because$ Unit digit of the number = 2

$\therefore$ Unit digit of the cube of the number = 8

(viii) 53

$\because$ Unit digit of the number = 3

$\therefore$ Unit digit of the cube of the number = 7

Q.2 Consider the following pattern.

$$2^3 - 1^3 = 1 + 2 \times 1 \times 3$$
$$3^3 - 2^3 = 1 + 3 \times 2 \times 3$$
$$4^3 - 3^3 = 1 + 4 \times 3 \times 3$$

Using the above pattern, find the value of the following :

(i) $7^3 - 6^3$ (ii) $12^3 - 11^3$ (iii) $20^3 - 19^3$ (iv) $51^3 - 50^3$

Sol. (i) $7^3 - 6^3 = 1 + 7 \times 6 \times 3 = 1 + 126 = 127$

(ii) $12^3 - 11^3 = 1 + 12 \times 11 \times 3 = 1 + 396 = 397$

(iii) $20^3 - 19^3 = 1 + 20 \times 19 \times 3 = 1 + 1140 = 1141$

(iv) $51^3 - 50^3 = 1 + 51 \times 50 \times 3 = 1 + 7650 = 7651$

Q.3 Which of the following are perfect cubes ?

(1) 400 (2) 3375 (3) 8000 (4) 15625

(5) 9000 (6) 6859 (7) 2025 (8) 10648

Sol. (1) 400

2	400
2	200
2	100
2	50
2	25
5	5
	1

By prime factorisation, $400 = 2 \times 2 \times 2 \times 2 \times 5 \times 5$

In the above factorisation, 2 and 5×5 remain after grouping 2's in triplets.

Therefore, 400 is not a perfect cube.

(2) **3375**

3	3375
3	1125
3	375
5	125
5	25
5	5
	1

By prime facorisation,

$3375 = \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$ (grouping the factors in triplets)

$= 3^3 \times 5^3$ (By laws of exponents)

$= (3 \times 5)^3 = 15^3$, which is a perfect cube.

Therefore, 3375 is a perfect cube.

(3) **8000**

2	8000
2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
5	5
	1

By prime factorisation,

$8000 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5}$ (grouping the factors in tripletes)

$= 2^3 \times 2^3 \times 5^3$ (by laws of exponents)

$= (2 \times 2 \times 5)^3 = 20^3$, which is a perfect cube.

Hence, 8000 is a perfect cube.

(4) **15625**

5	15625
5	3125
5	625
5	125
5	25
5	5
	1

By prime factorisation,

$15625 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$ (grouping the factors in triplets)

$= 5^3 \times 5^3$ (by laws of exponents)

$= (5 \times 5)^3$

$= 25^3$, which is a perfect cube.

Therefore, 15625 is a perfect cube.

(5) **9000**

2	9000
2	4500
2	2250
3	1125
3	375
5	125
5	25
5	5
	1

By prime factorisation,

$9000 = \underline{2 \times 2 \times 2} \times 3 \times 3 \times \underline{5 \times 5 \times 5}$ (grouping the factors in triplets)

In the above factorisation, 3×3 remain after grouping 2's and 5's in triplets.

Therefore, 9000 is not a perfect cube.

(6) **6859**

19	6859
19	361
19	19
	1

By prime factorisatin,

$6859 = \underline{19 \times 19 \times 19}$ (grouping the factors in triplets)

$= 19^3$, which is a perfect cube. (by laws of exponents)

Therefore, 6859 is a perfect cube.

(7) **2025**

3	2025
3	675
3	225
3	75
5	25
5	5
	1

By prime factorisatin,

$2025 = \underline{3 \times 3 \times 3} \times 5 \times 5$ (grouping the factors in triplets)

In the above factorisation, 3 and 5×5 remain after grouping 3's in triplets.

Therefore, 2025 is not a perfect cube.

(8) **10648**

2	10648
2	5324
2	2662
11	1331
11	121
	11

By prime factorisation,

10648 $= 2 \times 2 \times 2 \times 11 \times 11 \times 11$ (grouping the factors in triplets)

$= 2^3 \times 11^3$ (by laws of exponents)

$= (2 \times 11)^3$

$= 22^3$, which is a perfect cube,

Therefore, 10648 is a perfect cube.

Q.4 Check which of the following are perfect cubes.

(i)	**2700**	**(ii)**	**16000**	**(iii)**	**64000**	**(iv)**	**900**
(v)	**125000**	**(vi)**	**36000**	**(vii)**	**21600**	**(viii)**	**10,000**
(ix)	**27000000**	**(x)**	**1000.**				

What pattern do you observe in these perfect cubes ?

Sol. (i) 2700

2	2700
2	1350
3	675
3	225
3	75
5	25
5	5
	1

By prime factorisatin,

2700 $= 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5$ (grouping the factors in triplets)

In the above factorisation, 2×2 and 5×5 remain after grouping 3's in triplets.

Therefore, 2700 is not a perfect cube.

(ii) 16000

2	16000
2	8000
2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
5	5
	1

By prime factorisation

16000 $= \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2 \times 2} \times \underline{5 \times 5 \times 5}$ (grouping the factors in triples)

In the above factors, 2 remains after grouping in triplets.

Therefore, 16000 is not a perfect cube.

(iii) **64000**

2	64000
2	32000
2	16000
2	8000
2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
5	5
	1

By prime factorisation,

64000 $= \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5}$ (grouping the factors in triplets)

$= 2^3 \times 2^3 \times 2^3 \times 5^3$

$= (2 \times 2 \times 2 \times 5)^3$ (by laws of exponents)

$= 40^3$, which is a perfect cube.

Therefore, 64000 is a perfect cube.

(iv) **900**

2	900
2	450
3	225
3	75
5	25
5	5
	1

By prime factorisation,

900 $= 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5$

In the above factorisation, 2×2, 3×3 and 5×5 remain when we try to group the factors in triplets.

Therefore, 900 is not a perfect cube.

(v) **125000**

2	125000
2	62500
2	31250
5	15625
5	3125
5	625
5	125
5	25
5	5
	1

By prime factorisation,

$125000 = \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5} \times \underline{5 \times 5 \times 5}$ (grouping the factors in triplets)

$= 2^3 \times 5^3 \times 5^3$ (by laws of exponents)

$= (2 \times 5 \times 5)^3$ (by laws of exponents)

$= 50^3$, which is a perfect cube.

Therefore, 125000 is a perfect cube.

(vi) 36000

2	36000
2	18000
2	9000
2	4500
2	2250
3	1125
3	375
5	125
5	25
	5

By prime factorisation,

$36000 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2} \times \underline{3 \times 3} \times \underline{5 \times 5 \times 5}$ (grouping the factors in triplets)

In the above factorisation, 2 × 2 and 3 × 3 remain after grouping 2's and 5's in triplets.

Therefore, 36000 is not a perfect cube.

(vii) 21600

2	21600
2	10800
2	5400
2	2700
2	1350
3	675
3	225
3	75
5	25
5	5
	1

By prime factorisation,

$21600 = \underline{2 \times 2 \times 2} \times 2 \times 2 \times \underline{3 \times 3 \times 3} \times 5 \times 5$ (grouping the factors in triplets)

In the above factorisation, 2 × 2 and 5 × 5 remain after grouping 2's and 3's in triplets.

Therefore, 21600 is not a perfect cube.

(viii) 10000

2	10000
2	5000
2	2500
2	1250
5	625
5	125
5	25
	5

By prime factorisation,

$10,000 = \underline{2 \times 2 \times 2} \times 2 \times \underline{5 \times 5 \times 5} \times 5$ (grouping the factors in triplets)

In the above factorisation, 2 and 5 remain after grouping 2's and 5's in triplets.

Therefore, 10000 is not a perfect cube.

(ix) 27000000

2	27000000
2	13500000
2	67500000
2	3375000
2	1687500
2	843750
3	421875
3	140625
3	46875
5	15625
5	3125
5	625
5	125
5	25
5	5
	1

By prime factorisation,

$27000000 = \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5} \times \underline{5 \times 5 \times 5}$

(grouping the factors triplets)

$= 2^3 \times 2^3 \times 3^3 \times 5^3 \times 5^3$

$= (2 \times 2 \times 3 \times 5 \times 5)^3$ (by laws of exponents)

$= 300^3$, which is a perfect cube.

Therefore, 27000000 is a perfect cube.

(x) **1000**

2	1000
2	500
2	250
5	125
5	25
	5

By prime factorisation,

$1000 = \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5}$ (grouping factorisation triplets)

$= 2^3 \times 5^3 = (2 \times 5)^3$ (by laws of exponents)

$= 10^3$, which is a perfect cube.

Therefore, 1000 is a perfect cube. We observe the following pattern in these perfect cubes.

(i) If in the end of a number, the number of zeros is not 3 or a multiple of 3, then that number cannot be a perfect cube.

(ii) If in the end of a number, the number of zeros is 3 or a multiple of 3, then that number may be a perfect cube.

Thus, the number of zeros at the end of a perfect cube must essentially be 3 or a multiple of 3, failing which the number cannot be a perfect cube.

Q.5 State true or false : for any integer m, $m^2 < m^3$. Why ?

Sol. False, if m is a negative integer and true, if m is a positive integer (natural number).

Verification. Let us take a negative integer m = – 1. Then,

$m^2 = (-1)^2 = (-1) \times (-1) = 1$

$m^3 = (-1)^3 = (-1) \times (-1) \times (-1) = -1$

Clearly, $m^2 > m^3$

Hence, the given statement is false if m is a negative integer.

We get the same inference for m = – 2, – 3, – 4, ...,etc.

Again, let us take a positive integer (natural number) m = 2. Then,

$m^2 = (2)^2 = 2 \times 2 = 4$

$m^3 = (2)^3 = 2 \times 2 \times 2 = 8$

Clearly, $m^2 < m^3$

Hence, the given statement is true if m is a positive integer (natural number).

We get the same inference for m = 3, 4, 5, ...etc.

CONCEPT APPLICATION LEVEL - II

SECTION - A

- **FILL IN THE BLANKS :**

Q.1 Is 64000 a perfect cube? __________

Q.2 The smallest natural number by which 9 must be multiplied to get a perfect cube is________

Q.3 The cube root of (–8000) is _________ .

Q.4 The cube root of $-(8 \times 27)$ is _________.

Q.5 The cube root of (27×64) is __________

Q.6 The value of $\sqrt[3]{4^3 \times 6^3}$ is __________

Q.7 The value of $\sqrt[3]{\frac{-8}{125}}$ is __________

Q.8 $\sqrt[3]{\frac{3.43}{10}} =$ __________.

Q.9 $\sqrt[3]{a^6 \times b^9} =$ __________.

Q.10 $\sqrt[3]{0.125} + \sqrt[3]{0.729} =$ __________.

Q.11 $\sqrt[3]{-m^6} =$ __________.

SECTION - B

- **Mark true (T) or false (F) for the following statements.**

Q.1 If n is a multiple of 2, then n^3 is also a multiple of 2.

Q.2 If n is not a multiple of 2, then n^3 is also not a multiple of 2.

Q.3 If n ends in 3, then n^3 ends in 7.

Q.4 If n ends in 5, then n^3 ends in 25.

Q.5 A perfect cube can end with even number of zeroes.

SECTION - C

- **Multiple choice question with one correct answers**

Q.1 Cube of an odd natural number is

(A) an even natural number (B) an odd natural number
(C) a prime number (D) none of these

Q.2 Cube of an even natural number is

(A) an even natural number (B) an odd natural number
(C) a prime number (D) none of these

Q.3 Cube root of a negative number is
(A) a negative number (B) a positive number
(C) sometimes negative, sometimes positive (D) none of these

Q.4 Cube root of the product of two negative numbers is
(A) a negative number (B) a positive number
(C) sometimes negative, sometimes positive (D) none of these

Q.5 For a non-zero integer x, x^3 is
(A) always less than x^2
(B) always greater than x^2
(C) sometimes less and sometimes greater than x^2
(D) none of these

Q.6 What is the value of $\sqrt[3]{0.000064}$?
(A) 0.4 (B) 0.08 (C) 0.04 (D) 0.16

Q.7 What is the value of $\sqrt[3]{\sqrt{441}+\sqrt{16}+\sqrt{4}}$
(A) 3 (B) 5 (C) 7 (D) None

Q.8 The smallest number by which 3600 must be multiplied to make it a perfect cube
(A) 40 (B) 60 (C) 20 (D) 15

Q.9 $\sqrt[3]{-1}$ = ?
(A) –1 (B) 1 (C) –1/3 (D) None of these

Q.10 $\sqrt[3]{\dfrac{72.9}{0.4096}}$ is equal to
(A) 0.5625 (B) 5.625 (C) 182 (D) 13.6

Q.11 The digit in the unit's place in the cube root of 21952 is
(A) 8 (B) 6 (C) 4 (D) 2

Q.12 If the cube root of 175616 is 56, then the value of $\sqrt[3]{175.616}+\sqrt[3]{0.175616}+\sqrt[3]{0.000175616}$ is equal to
(A) 0.168 (B) 62 – 16 (C) 6.216 (D) 6.116

Q.13 $\sqrt{\sqrt[3]{0.004096}}$ is equal to
(A) 4 (B) 0.4 (C) 0.04 (D) 0.004

Q.14 The value of $\sqrt[3]{(-343)\times(512)}$ is
(A) 56 (B) –56 (C) 65 (D) –65

Q.15 The volumes of two cubes are in the ratio of 343 : 1331, the ratio of their edges is
(A) 7 : 10 (B) 7 : 11 (C) 7 : 12 (D) None of these

Q.16 The smallest natural number by which 32 must be multiplied to get a perfect cube is
(A) 16 (B) 4 (C) 2 (D) 8

Q.17 The smallest natural number by which 32 must be divided to get a perfect cube is
(A) 16 (B) 4 (C) 2 (D) 8

Q.18 $\sqrt[3]{8\times 64} = ?$
(A) 12 (B) 16 (C) 8 (D) 24

Q.19 If the volume of a cube is 512 cm^3, then the length of its side is
(A) 8 cm (B) 9 cm (C) 7 cm (D) 6 cm

Q.20 The cube root of $\sqrt[3]{-125}$ is
(A) 5 (B) –5 (C) 25 (D) None of these

Q.21 The value of $\sqrt[3]{-2^3}$ is
(A) -2^3 (B) –2 (C) 2^3 (D) 2

Q.22 Which of the following statements is true?
(A) Cube of an even number is odd
(B) Cube of a number ending with 3 ends with 9.
(C) Cube of a number ending with 0 has three 0's at its extreme right
(D) Cube of a 2-digit number may be a three digit number

Q.23 The cube of 70 is
(A) 49000 (B) 490000 (C) 343000 (D) 34300

Q.24 The cube of (–5) is
(A) 25 (B) –125 (C) 125 (D) –25

Q.25 The cube of $\left(2-\frac{1}{3}\right)$ is
(A) $8-\frac{1}{27}$ (B) $\frac{125}{27}$ (C) $\frac{25}{9}$ (D) $\frac{343}{27}$

Q.26 The cube root of (–0.000001) is
(A) –0.1 (B) –0.01 (C) –0.001 (D) –0.0001

Q.27 The value of $\sqrt[3]{343}\times\sqrt[3]{-27}$ is
(A) 21 (B) –19 (C) 19 (D) –21

Q.28 $\sqrt[3]{\dfrac{-a^6 \times b^3 \times c^{21}}{c^9 \times a^{12}}} =$

(A) $\dfrac{-bc^3}{a^2}$ (B) $\dfrac{bc^4}{a^2}$ (C) $\dfrac{-ab^4}{c^2}$ (D) $\dfrac{-bc^4}{a^2}$

Q.29 The cube of the number p is 16 times the number. Then find p where $p \neq 0$ and $p \neq -4$.

(A) 4 (B) 3 (C) 8 (D) 2

Q.30 The cube of a number x is nine times of x, then find x, $x \neq 0$ and $x \neq -3$

(A) 8 (B) 2 (C) 4 (D) 3

Q.31 The digit in the units place for the cube of the number 1234568 is __________.

(A) 8 (B) 2 (C) 4 (D) 6

Q.32 Which of the following is not a perfect square?

(A) 16384 (B) 23857 (C) 18496 (D) 11025

Q.33 $\sqrt[3]{\dfrac{3^6 \times 4^3 \times 2^6}{8^9 \times 2^3}} =$ __________.

(A) $\dfrac{3}{8}$ (B) $\dfrac{9}{8}$ (C) $\dfrac{3}{64}$ (D) $\dfrac{9}{64}$

Q.34 The cube root of the number 10648 is __________.

(A) 42 (B) 38 (C) 28 (D) 22

Q.35 The cube of a number ending in 3, ends in __________.

(A) 3 (B) 7 (C) 9 (D) Cannot say

Q.36 Find the value of $\sqrt[3]{6075} \times \sqrt[3]{88935} \times \sqrt[3]{9625}$.

(A) 17355 (B) 17255 (C) 17315 (D) 17325

ANSWER KEY

CONCEPT APPLICATION LEVEL - II

SECTION - A

Q.1	40	Q.2	$(3)^3$	Q.3	– 20	Q.4	– 6	Q.5	12	Q.6	24	Q.7	$\frac{-2}{5}$
Q.8	0.7	Q.9	$a^2 \times b^3$	Q.10	1.4	Q.11	$-m^2$						

SECTION - B

Q.1	True	Q.2	True	Q.3	True	Q.4	False	Q.5	True

SECTION - C

Q.1	B	Q.2	A	Q.3	A	Q.4	B	Q.5	C	Q.6	C	Q.7	A
Q.8	B	Q.9	A	Q.10	B	Q.11	A	Q.12	C	Q.13	B	Q.14	B
Q.15	B	Q.16	C	Q.17	B	Q.18	C	Q.19	A	Q.20	B	Q.21	B
Q.22	C	Q.23	C	Q.24	B	Q.25	B	Q.26	B	Q.27	D	Q.28	D
Q.29	A	Q.30	D	Q.31	B	Q.32	B	Q.33	D	Q.34	D	Q.35	B
Q.36	D												