5 ALGEBRAIC EXPRESSIONS AND IDENTITIES

THEORY

5.1 INTRODUCTION

Algebra is the branch of mathematics concerning the study of the rules of operations and relations. Elementary algebra is the most basic form of algebra. It is taught to students who are presumed to have no knowledge of mathematics beyond the basic principles of arithmetic. In arithmetic, only numbers and their arithmetical operations (such as +, –, ×, ÷) occur. In algebra, numbers are often denoted by symbols (such as a, x or y). This is useful becuase :

A polynomial is an expression that is constructed from one or more variables and constants, using only the operations of addition, subtraction, and multiplication.

for example, $x^2 + 2x - 3$ is a polynomial in the single variable x.

An important class of problems in algebra is factorization of polynomials, that is expressing a given polynomial as a product of other polynomials. The above polynomial can be factored as $(x - 1)(x + 3)$.

5.2 ALGEBRAIC EXPRESSIONS

The branch of mathematics which deals with numbers is called Arithmetic. Algebra can be considered as generalisation of arithmetic, where we use letter in place of numbers.

- **Constants:** A symbol having a fixed numerical value is called a constant.
  For example, 8, –6, 5/7, π etc are all constants

- **Variables:** A symbol which may be assinged different numerical values is known as a variable.
  For example, circumference of a circle is given by
  $$c = 2\pi r$$
  Here, 2 and π are constants, while c and r are variables.

- **Terms of an Algebraic Expression:** The several parts of an algebraic expression separated by + or – operations are called the terms of the expression.

  For example : $4 + 9x - 5x^2y + \frac{3}{5}xy$ is an algebraic expression containing four terms, namely, 4, $9x$, $-5x^2y$ and $3/5\ xy$.

- **Factors of term :**
  Ex. The term $9y^2$ is a product of 9, y and y. Thus 9, y and y are the factors of $9y^2$.

- **Coefficient of a term :** Consider a algebraic expression $3x^2 + 5x + 6$. In $3x^2 + 5x + 6$, $3x^2$ is first term, 5x is second term and 6 is the third term. In the first term $3x^2$, 3 is called numerical coefficient and x^2 is called literal coefficient. Similarly in the second term 5x, 5 is called numerical coefficient and x is called literal coefficient.

- **Like terms :** In any algebraic expression, the terms having the same literal coefficients are called like terms.

  For example : $6x^3$, $-x^3$, $2x^3$ and $\frac{1}{4}x^3$ are like terms.

- **Unlike terms :** In any algebraic expression, the terms having different literal coefficient are called unlike terms.
  For example : 7x, x^2, $2x^3$ and $15x^4$ are unlike terms.

- **Algebraic Expressions:** A combination of constants and variables, connected by operations +, –, × and ÷ is known as an algebraic expressions.

5.2.1 Type of Algebraic Expressions

- **Monomial** - one term. **Ex. :** $3p^2q^2$
- **Binomial -** two terms. **Ex. :** $3x + 4y$
- **Trinomial -** three terms. **Ex. :** $x^2 + y^2 + 6$

5.3 POLYNOMIALS

An algebraic expression f(x) of the form $f(x) = a_0 + a_1x + a_2x^2 + ...+ a^nx^n$; where $a_0, a_1, a_2, ..., a_n$ are real numbers and all the indices of variable x are non-negative integers, is called a polynomial in variable x and the highest indices n is called the degree of the polynomial, if $a_n \neq 0$. Here, a_0, a_1x, a_2x^2 and a_nx^n are called the terms of the polynomial and $a_0, a_1, a_2..............a_n$ are called various coefficients of the polynomial f(x). A polynomial in x is said to be in standard form when the terms are written either in increasing order or in decreasing order of the indices of x in various terms.

For example : $x^2 - a^2$, $ax^2 + bx + c$, $x^3 + 3x^2 + 3x + 1$, $y^3 - 7y + 6$ etc. are the polynomials written in their standard form.

- **Simplest form of Polynomial :** A polynomial is said to be in simplest form when no two of the polynomial are like terms.
  For example : $3x^2 + 4x + 2$.

- **Standard form of Polynomial :** When a polynomial is written in either ascending or descending power of variable.
  For example : $x^3 - 2x^2 + 3x - 6$

- **Degree of a Polynomial in One Variable :** In case of a polynomial in one variable, the highest power of the variable is called the degree of the polynomial.

- **Degree of a Polynomial in Two or More Variables :** In case of polynomials in more than one variable, the sum of the powers of the variables in each term is taken up and the highest sum so obtained is called the degree of the polynomial.

5.3.1 Types of polynomials

Polynomials can be classified on the basis of number of terms and on the basis of degree.

On the basis of degree :

(i) **Zero polynomial :** A polynomial f(x) = 0 is called zero polynomial. Its degree is not defined.

(ii) **Constant polynomial :** A polynomial of degree zero is called a constant polynomial.
For example : 2, –5, 7. Every real number is a constant polynomial

(iii) **Linear polynomial :** A polynomial of degree 1 is called a linear polynomial.
For example : $9x + 5$ is a linear polynomial in x.
$x + y + 4$ is a linear polynomial in x and y.

(iv) **Quadratic polynomial :** A polynomial of degree 2 is called a quadratic polynomial.
For example : $2y^2 - 8y + 5$ is a quadratic polynomial in y.
$2xy + 5x + 3y + 4$ is a quadratic polynomial in x and y.

(v) **Cubic polynomial :** A polynomial of degree 3 is called a cubic polynomial.
For example : $2x^3 - 3x^2 + 5x + 1$ is a cubic polynomial in x
$2x^2y + 5xy^2 + 8$ is a cubic polynomial in x and y.

(vi) **Biquadratic polynomial :** A polynomial of degree 4 is called a biquadratic polynomial.
For example : $z^4 + 6z^3 + 10z^2 + 6z + 1$ is biquadratic polynomial in z.
$3x^2yz + 4xy^2z + 5xyz^2$ is biquadratic polynomial in, x, y and z.

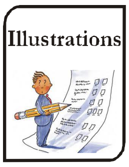


Illustration 1

Find the degree of each of the following polynomials.

(i) $2x^3 + x^2 - x + 4$ (ii) $x + 4 - 3x^3 + x^4$ (iii) 10

Solution

(i) **$2x^3 + x^2 - x + 4$:** The highest power term is $2x^3$. The power of variable in this term is 3. So the degree of given polynomial is 3.

(ii) **$x + 4 - 3x^3 + x^4$:** The highest power term is x^4.
$\therefore$ Degree of polynomial is 4.

(iii) **10 :** 10 is a constant polynomial. It can be written as $10.\ x^0$ (As $x^0 = 1$) where x is any variable. Highest power of variable is 0 (zero), so the degree of constant polynomial is '0'.
'0' itself is a constant polynomial.

Illustration 2

Classify the following polynomials as linear, quadratic or cubic polynomials.

(i) $10x^2$ (ii) y (iii) $1 + z$ (iv) $y + y^3$ (v) $x^2 + x + 5$

Solution

(i) $10x^2$: Degree of polynomial $10x^2$ is '2' so it is a quadratic polynomial.
(ii) y : Degree of polynomial is '1' so the polynomial is a quadratic polynomial.
(iii) $1 + z$: Degree of polynomial is '1'. It is a linear polynomial.
(iv) $y + y^3$: Degree of polynomial is '3'. It is a cubic polynomial.
(v) $x^2 + x + 5$: Degree of polynomial is '2'. It is a quadratic polynomial.

5.4 ADDITION AND SUBTRACTION OF ALGEBRAIC EXPRESSIONS

For addition or subtraction of two or more than two algebraic expressions, we first collect like terms and then find the sum or difference of coefficients of these terms.

Illustration 3

Add : $l^2 + m^2, m^2 + n^2, n^2 + l^2$ and $2lm + 2mn + 2nl$

Solution

Required sum

$= l^2 + m^2 + m^2 + n^2 + n^2 + l^2 + 2lm + 2mn + 2nl$

$= (l^2 + l^2) + (m^2 + m^2) + (n^2 + n^2) + 2lm + 2mn + 2nl$ (Collecting like terms)

$= 2l^2 + 2m^2 + 2n^2 + 2lm + 2mn + 2nl$

$= 2\,(l^2 + m^2 + n^2 + lm + mn + nl)$

Illustration 4

Subtract : $4p^2q - 3pq + 5pq^2 - 8p + 7q - 10$ from $18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q$.

Solution

We have

$18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q - (4p^2q - 3pq + 5pq^2 - 8p + 7q - 10)$

$= 18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q - 4p^2q + 3pq - 5pq^2 + 8p - 7q + 10$

$= (18 + 10) + (8p - 3p) + (-11q - 7q) + (5pq + 3pq) + (-2pq^2 - 5pq^2) + (5p^2q - 4p^2q)$

(Collecting like terms)

$= 28 + 5p - 18q + 8pq - 7pq^2 + p^2q$

Illustration 5

Subtract the sum of $3l - 4m - 7n^2$ and $2l + 3m - 4n^2$ from the sum of $9l + 2m - 3n^2$ and $-3l + m + 4n^2$.

Solution

Sum of $3l - 4m - 7n^2$ and $2l + 3m - 4n^2$

$= 3l - 4m - 7n^2 + 2l + 3m - 4n^2$

$= (3l + 2l) + (3m - 4m) + (-7n^2 - 4n^2)$

$= 5l - m - 11n^2$

Sum of $9l + 2m - 3n^2$ and $-3l + m + 4n^2$

$= 9l + 2m - 3n^2 - 3l + m + 4n^2$

$= (9l - 3l) + (2m + m) + (-3n^2 + 4n^2)$

$= 6l + 3m + n^2$

Required difference

$= 6l + 3m + n^2 - (5l - m - 11n^2)$

$= 6l + 3m + n^2 - 5l + m + 11n^2$

$= (6l - 5l) + (3m + m) + (n^2 + 11n^2)$

$= l + 4m + 12n^2$.

5.5 MULTIPLICATION OF ALGEBRAIC EXPRESSIONS

We know that the product of two integers with the same sign is positive and the product of two integers with the opposite signs is negative.

i.e., (+) × (+) = (+), (–) × (–) = (+)

(+) × (–) = (–), (–) × (+) = (–)

Also, from the chapters of exponents, we know that

(i) $x^m \times x^n = x^{m+n}$

(ii) $(x^m)^n = x^{mn}$, where x, m, n are non-zero integers.

While multiplying algebraic expressions, we shall make use of these concepts.

5.5.1 Multiplying Two Monomials

Consider, $2x^3 \times 3x$

We known, $2x^3 = 2 \times x \times x \times x$ and $3x = 3 \times x$

So, $2x^3 \times 3x = (2 \times x \times x \times x) \times (3 \times x)$

$= (2 \times 3) \times (x \times x \times x \times x)$

$= 6x^4$

How did we perform the above multiplication ?

There are three steps :

(i) Multiply the coefficients of both the monomials.

(ii) Multiply the variables.

(iii) Multiply the above two results.

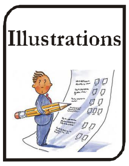


Illustration 6

Multiply each of the following :

(i) $6xy$ and $5x^2y^2z$ (ii) $-7x^2yz$ and $\frac{2}{3}xy^3$ (iii) $\frac{-8}{5}a^2bc^3$ and $\frac{-3}{4}ab^2x$

Solution

(i) $6xy \times 5x^2y^2z = (6 \times 5) \times (x \times x^2) \times (y \times y^2) \times z$

$= (6 \times 5) \times x^{1+2} \times y^{1+2} \times z$ (Using $x^m \times x^n = x^{m+n}$)

$= 30x^3y^3z$

(ii) $(-7x^2yz) \times \left(\frac{2}{3}xy^3\right) = \left(-7 \times \frac{2}{3}\right)(x^2 \times x) \times (y \times y^3) \times z$

$= \frac{-14}{3}x^{2+1}y^{1+3}z$ (Using $x^m \times x^n = x^{m+n}$)

$= \frac{-14}{3}x^3y^4z$

(iii) $\left(\frac{-8}{5}a^2bc^3\right) \times \left(\frac{-3}{4}ab^2x\right) = \left(\frac{-8}{5} \times \frac{-3}{4}\right) \times (a^2 \times a) \times (b \times b^2) \times c^3 \times x$

$= \frac{6}{5}a^{2+1}b^{1+2}c^3x = \frac{6}{5}a^3b^3c^3xc$

5.5.2 Multiplying Three or More Monomials

The following rule can be used for multiplying any number of monomials.

(i) The coefficient of the product of the given monomials is the product of the coefficients of these monomials.

(ii) The exponent of each literal is the sum of the exponents of this literal in the given monomials.

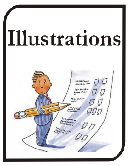


Illustration 7

Find the product of

(i) $5xyz, 10x^2y^2z^2, -3x^2y^3z^4$ and $6x^2y^2z^5$

(ii) $-8xyz, 4x^3y^2z^2, 3x^2y^2z^2$ and $-2yz$

Solution

(i) $5xyz \times 10x^2y^2z^2 \times (-3x^2y^3z^4) \times 6x^2y^2z^5$

$= 5 \times 10 \times (-3) \times 6 \times (x \times x^2 \times x^2 \times x^2) \times (y \times y^2 \times y^3 \times y^2) \times (z \times z^2 \times z^4 \times z^5)$

$= -900 \times x^{1+2+2+2}\ y^{1+2+3+2}\ z^{1+2+4+5}$

$= -900\ x^7\ y^8\ z^{12}$

(ii) $(-8xyz) \times 4x^3y^2z^2 \times 3x^2y^2z^2 \times (-2yz)$

$= -8 \times 4 \times 3 \times (-2) \times (x \times x^3 \times x^2) \times (y \times y^2 \times y^2 \times y) \times (z \times z^2 \times z^2 \times z)$

$= 192 \times x^{1+3+2} \times y^{1+2+2+1} \times z^{1+2+2+1}$

$= 192\ x^6y^6z^6$

Illustration 8

Obtain the volume of the rectangular box whose length, breadth and height are xy, $2x^2y$ and $2xy^2$ respectively.

Solution

We know that volume of a rectangular box is the product of its length, breadth and height.

$\therefore$ Volume of the box $= xy \times 2x^2y \times 2xy^2$

$= 2 \times 2 \times (x \times x^2 \times x) \times (y \times y \times y^2)$

$= 4 \times x^4 \times y^4 = 4x^4y^4.$

Illustration 9

Find the value of $5a^6 \times (-10ab^2) \times \left(-\frac{1}{25}a^2b^3\right)$ for $a = 1$ and $b = 2$.

Solution

We have,

$$5a^6 \times (-10ab^2) \times \left(-\frac{1}{25}a^2b^3\right) = 5 \times (-10) \times \left(-\frac{1}{25}\right) \times (a^6 \times a \times a^2) \times (b^2 \times b^3)$$

$$= 2 \times a^{6+1+2} \times b^{2+3} = 2a^9b^5$$

Putting $a = 1$ and $b = 2$, we have

$2a^9b^5 = 2 \times (1)^9 \times (2)^5 = 2 \times 1 \times 32 = 64$

5.5.3 Multiplying a Monomial by a Bionomial

In the case of integers we use the distributive property of multiplication over addition for simpifying the products like x × (y + z). Similarly, in case of algebraic expressions, we use this property. If A, B and C are three monomials, then A × (B + C) = A × B + A × C

Illustration 10

Find the product of $9x^2y$ and (x + 2y).

Solution

$$9x^2y \times (x + 2y) = (9x^2y \times x) + (9x^2y \times 2y)$$
$$= 9x^3y + 9 \times 2 \times (x^2y \times y)$$
$$= 9x^3y + 18x^2y^2$$

The method of multiplying a monomial and a binomial discussed above is called the **horizontal** or **row method**, because the working is done horizontally. We can also multiply a monomial and a binomial vertically i.e., from top to bottom in columns, which is called **vertical method.** Observe the following example :

e.g. Multiply 9xy and $3xy + 5y^2$.

Solution

$$\begin{array}{r} 3xy + 5y^2 \\ \times \quad 9xy \\ \hline 27x^2y^2 + 45xy^3 \\ \hline \end{array}$$

$9xy \times 3xy$ $\quad$ $9xy \times 5y^2$

[Multiply 3xy and $5y^2$ by 9xy separately and add them]

5.5.4 Multiplying a Monomial by a Trinomial

If A, B, C and D are four monomials, then A × (B + C + D) = A × B + A × C + A × D

Illustration 11

Multiply 3*l* and *l* – 4m + 5n

Solution

$$3l\,(l - 4m + 5n) = 3l \times l - 3l \times 4m + 3l \times 5n$$
$$= 3l^2 - 12lm + 15\,ln$$

5.5.5 Multiplying a Binomial by a Binomial

Multiplication of two binomials can be performed using distributive property of multiplication over addition twice. If A, B, C and D are four monomials, then

$$(A + B) \times (C + D) = A \times (C + D) + B \times (C + D)$$
$$= A \times C + A \times D + B \times C + B \times D$$
$$= AC + AD + BC + BD.$$

This multiplication can also be performed by column method, as follows :

$$\begin{array}{rl} A + B & \\ \times\ \ C + D & \\ \hline AD + BD & \text{[Multiply (A + B) by D]} \\ AC + BC\quad\ \ & \text{[Multiply (A + B) by C]} \\ \hline AC + AD + BC + BD & \\ \hline \end{array}$$

5.5.6 Multiplying a Binomial by a Trinomial

Multiplication of a binomial and a trinomial can be performed using distributive property of multiplication over addition. If A, B, C, D and E are five monomials, then

$(A + B) \times (C + D + E) = A \times (C + D + E) + B \times (C + D + E)$

$= AC + AD + AE + BC + BD + BE.$

This multiplication can also be performed by column method, al follows :

$$\begin{array}{l} \quad\ \ C + D + E \\ \qquad\ \times\ A + B \\ \hline BC + BD + BE \\ AC + AD + AE \\ \hline AC + AD + AE + BC + BD + BE \\ \hline \end{array}$$

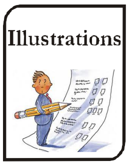


Illustration 12

Multiply the following :

(i) (9a + 6b) and (6a + 9b)

(ii) $(2x^2y + 3y^2)$ and $(8y - 4x^2)$

Solution

(i) Using distributive property of multiplication over addition :

$(9a + 6b) \times (6a + 9b) = 9a \times (6a + 9b) + 6b \times (6a + 9b)$

$= 9a \times 6a + 9a \times 9b + 6b \times (6a + 9b)$

$= 54a^2 + 81ab + 36ab + 54b^2$

$= 54a^2 + 117ab + 54b^2$

By column method :

$$\begin{array}{r} 9a + 6b \\ \times\quad 6a + 9b \\ \hline 81ab + 54b^2 \\ 54a^2 + 36ab\qquad\quad \\ \hline 54a^2 + 117ab + 54b^2 \\ \hline \end{array}$$

(ii) Using distributive property of multiplication over addition :

$(2x^2y + 3y^2) \times (8y - 4x^2)$

$= 2x^2y \times (8y - 4x^2) + 3y^2 \times (8y - 4x^2)$

$= 2x^2y \times 8y + 2x^2y \times (-4x^2) + 3y^2 \times 8y + 3y^2 \times (-4x^2)$

$= 16x^2y^2 - 8x^4y + 24y^3 - 12x^2y^2$

$= 16x^2y^2 - 12x^2y^2 - 8x^4y + 24y^3$

$= 4x^2y^2 - 8x^4y + 24y^3$

Illustration 13

Subtract 3a(a + b + c) – 2b(a – b + c) from 4c(–a + b + c).

Solution

We have $3a(a + b + c) - 2b(a - b + c)$

$= (3a \times a + 3a \times b + 3a \times c) - (2b \times a - 2b \times b + 2b \times c)$

$= 3a^2 + 3ab + 3ac - 2ab + 2b^2 - 2bc$

$= 3a^2 + (3ab - 2ab) + 3ac - 2bc + 2b^2$

$= 3a^2 + ab + 3ac - 2bc + 2b^2$

$= 4c(-a + b + c)$

$= 4c \times (-a) + 4c \times b + 4c \times c$

$= -4ac + 4bc + 4c^2$

Now,

$\therefore$ $4c(-a + b + c) - \{3a(a + b + c) - 2b(a - b + c)\}$

$= -4ac + 4bc + 4c^2 - (3a^2 + ab + 3ac - 2bc + 2b^2)$

$= -4ac + 4bc + 4c^2 - 3a^2 - ab - 3ac + 2bc - 2b^2$

$= (-4ac - 3ac) + (4bc + 2bc) - ab + 4c^2 - 3a^2 - 2b^2$

$= -7ac + 6bc - ab - 3a^2 - 2b^2 + 4c^2$

Illustration 14

Simplify the following : $9x^2 + 7x(3x - 2y) + 5xy$

Solution

$9x^2 + 7x(3x - 2y) + 5xy$

$= 9x^2 + 21x^2 - 14xy + 5xy$

$= 30x^2 - 9xy.$

Illustration 15

Simplify : $9x^4(2x^3 - 5x^4) \times 5x^6(x^4 - 3x^2)$

Solution

$9x^4(2x^3 - 5x^4) \times 5x^6(x^4 - 3x^2)$

$= (9x^4 \times 2x^3 - 9x^4 \times 5x^4) \times (5x^6 \times x^4 - 5x^6 \times 3x^2)$

$= (18x^7 - 45x^8) \times (5x^{10} - 15x^8)$

$= 18x^7 \times (5x^{10} - 15x^8) - 45x^8 \times (5x^{10} - 15x^8)$

$= 18x^7 \times 5x^{10} - 18x^7 \times 15x^8 - 45x^8 \times 5x^{10} + 45x^8 \times 15x^8$

$= 90x^{17} - 270x^{15} - 225x^{18} + 675x^{16}$

$= -225x^{18} + 90x^{17} + 675x^{16} - 270x^{15}$

Illustration 16

Multiply : $(2x^2 - 3x + 5)$ by $(5x + 2)$

Solution

We have,

$(5x + 2) \times (2x^2 - 3x + 5)$

$= 5x \times (2x^2 - 3x + 5) + 2 \times (2x^2 - 3x + 5)$

$= 5x \times 2x^2 + 5x \times (-3x) + 5x \times 5 + 2 \times 2x^2 + 2 \times (-3x) + 2 \times 5$

$= 10x^3 - 15x^2 + 25x + 4x^2 - 6x + 10$

$= 10x^3 - 11x^2 + 19x + 10$

Illustration 17

Find the product : $\left(2x - \frac{1}{2}y\right)\left(\frac{3}{4}x - 10y + 8\right)$

Solution

$$\left(2x - \frac{1}{2}y\right)\left(\frac{3}{4}x - 10y + 8\right)$$

$$= 2x\left(\frac{3}{4}x - 10y + 8\right) - \frac{1}{2}y\left(\frac{3}{4}x - 10y + 8\right)$$

$$= 2x \times \frac{3}{4}x + 2x \times (-10y) + 2x \times 8 - \frac{1}{2}y \times \frac{3}{4}x - \frac{1}{2}y \times (-10y) - \frac{1}{2}y \times 8$$

$$= \frac{3}{2}x^2 - 20xy + 16x - \frac{3}{8}xy + 5y^2 - 4y$$

5.6 DIVISION OF POLYNOMIALS

Division is the inverse process of multiplication.

When we divide one expression by another, we find a third expression which when multiplied by the second gives the first, i.e., if $a \div b = x$ then $a = bx$. In $a \div b = x$, a is called the **Dividend**, b the **Divisor** and x is called the **Quotient**.

5.6.1 Rules of Signs in Divison

(a) When the dividend and the divisor have the same signs, the quotient has the plus sign.

(b) When the dividend and the divisor has opposite signs, the quotient has the negative sign.

The process of division may be divided in three cases :

(I) Division of a monomial by another monomial

(II) Division of a polynomial by a monomial

(III) Division of a polynomial by another polynomial.

We shall discuss these cases one by one.

CASE I : DIVISION OF A MONOMIAL BY ANOTHER MONOMIAL.

Illustration 18

Divide : (a) $15a^5$ by $5a^2$ (b) $36a^3b^5$ by $-12a^2b$ (c) $2x^3$ by $\sqrt{2}\,x$

Solution

(a) Quotient $= \frac{15a^5}{5a^2} = \left(\frac{15}{5}\right)\left(\frac{a^5}{a^2}\right) = 3a^3$

(b) Quotient $= \frac{36a^3b^5}{-12a^2b} = \left(\frac{36}{-12}\right)\left(\frac{a^3}{a^2}\right)\left(\frac{b^5}{b}\right) = -3ab^4$

(c) Quotient $= \frac{2x^3}{\sqrt{2}\,x} = \left(\frac{2}{\sqrt{2}}\right)\left(\frac{x^3}{x}\right) = \sqrt{2}x^2$.

CASE II : DIVISION OF A POLYNOMIAL BY A MONOMIAL

Divide each term of the polynomial by the monomial and then write the resulting quotients with their proper signs.

Illustrations

Illustration 19

Divide : (a) $-4x^3 - 6x^2 + 8x$ by $2x$ (b) $3x^4y - 4x^3y^2 + 5x^2y^3$ by $-6x^2y$

Solution

Dividing each term of the dividend by the divisor, we get

$$\text{(a) Quotient} = \frac{-4x^3 - 6x^2 + 8x}{2x} = \frac{-4x^3}{2x} - \frac{6x^2}{2x} + \frac{8x}{2x} = -2x^2 - 3x + 4$$

$$\text{(b) Quotient} = \frac{3x^4y - 4x^3y^2 + 5x^2y^3}{-6x^2y} = \frac{3x^4y}{-6x^2y} - \frac{4x^3y^2}{-6x^2y} + \frac{5x^2y^3}{-6x^2y}$$

$$= -\frac{x^2}{2} + \frac{2xy}{3} - \frac{5y^2}{6} = \frac{1}{2}x^2 + \frac{2}{3}xy - \frac{5}{6}y^2$$

CASE III : DIVISION OF A POLYNOMIAL BY ANOTHER POLYNOMIAL

It is advisable in this case to rearrange the dividend and the divisor in descending order.

Illustrations

Illustration 20

Divide $x^2 + 5x + 6$ by $x + 3$

Solution

Explanation :

$$
\begin{array}{r|l}
x+3 & x^2 + 5x + 6 \;(\, x + 2 \\
 & x^2 + 3x \\
 & -\quad - \\
\hline
 & \quad 2x + 6 \\
 & \quad 2x + 6 \\
 & \quad -\quad - \\
\hline
 & \quad 0 \\
\end{array}
$$

Quotient = $x + 2$

Remainder = 0

(a) Divide the first term (x^2) of the dividend by the first term (x) of the divisor. The result $x^2 \div x = x$ is the first term of the quotient.

(b) Multiply the divisor $x + 3$ by x, the first term of the quotient.

(c) Subtract the product $(x + 3)\,x = x^2 + 3x$ from the dividend $x^2 + 5x + 6$, i.e., $(x^2 + 5x + 6) - (x^2 + 3x) = 2x + 6$

(d) Proceed with this remainder $2x + 6$ as with the original dividend, i.e., divide $2x$ by x. The result $2x \div x = 2$ is the second term of the quotient.

(e) Multiply the divisor $(x + 3)$ by 2, the second term of thhe quotient. Now subtract $2(x + 3)$ from $2x + 6$, i.e., $2x + 6 - 2(x + 3) = 2x + 6 - 2x - 6 = 0$.
The remainder is 0.
Hence the required quotient = $x + 2$.

Illustration 21

Divide $3x^4 + 5x^3 - x^2 + 13x + 9$ by $3x + 2$ and verify that :
Dividend = Divisor × Quotient + Remainder

Solution : First divide $3x^4 + 5x^3 - x^2 + 13x + 9$ by $3x + 2$

$$
\begin{array}{l}
3x+2\,)\;3x^4+5x^3-x^2+13x+9\;(\;x^3+x^2-x+5\\
\quad\quad\;\; 3x^4+2x^3\\
\quad\quad\;\; -\quad\; -\\
\hline
\quad\quad\quad\quad\; 3x^3-x^2\\
\quad\quad\quad\quad\; 3x^3+2x^2\\
\quad\quad\quad\quad\; -\quad\; -\\
\hline
\quad\quad\quad\quad\; -3x^2+13x\\
\quad\quad\quad\quad\; -3x^2-2x\\
\quad\quad\quad\quad\; +\quad\; +\\
\hline
\quad\quad\quad\quad\quad\quad\; 15x+9\\
\quad\quad\quad\quad\quad\quad\; 15x+10\\
\quad\quad\quad\quad\quad\quad\; -\quad\; -\\
\hline
\quad\quad\quad\quad\quad\quad\; -1\\
\hline
\end{array}
$$

Quotient = $x^3 + x^2 - x + 5$; Remainder = -1

Now, Divisor × Quotient + Remainder

$= (3x + 2)(x^3 + x^2 - x + 5) - 1 = 3x(x^3 + x^2 - x + 5) + 2(x^3 + x^2 - x + 5) - 1$

$= 3x^4 + 3x^3 - 3x^2 + 15x + 2x^3 + 2x^2 - 2x + 10 - 1$

$= 3x^4 + 5x^3 - x^2 + 13x + 9$ = Dividend

AN IMPORTANT RESULT

When the remainder is zero, the divisor is a factor of the dividend.

Illustration 22

Find the value of a if $2x - 3$ is a factor of $2x^4 - x^3 - 3x^2 - 2x + a$.

Solution :

First we divide $2x^4 - x^3 - 3x^2 - 2x + a$ by $2x - 3$.

$$
\begin{array}{l}
\quad\quad\quad\quad\quad\quad\quad\quad\quad\; x^3+x^2-1\\
2x-3\,\overline{)\,2x^4-x^3-3x^2-2x+a}\\
\quad\quad\quad\; 2x^4-3x^3\\
\quad\quad\quad\; -\quad\; +\\
\hline
\quad\quad\quad\; 2x^3-3x^2\\
\quad\quad\quad\; 2x^3-3x^2\\
\quad\quad\quad\; -\quad\; +\\
\hline
\quad\quad\quad\quad\quad\quad\quad\; -2x+a\\
\quad\quad\quad\quad\quad\quad\quad\; -2x+3\\
\quad\quad\quad\quad\quad\quad\quad\quad +\quad -\\
\hline
\quad\quad\quad\quad\quad\quad\quad\quad a-3\\
\hline
\end{array}
$$

$\therefore$ $2x - 3$ is a factor of $2x^4 - x^3 - 3x^2 - 2x + a$ if, $a - 3 = 0$

Hence $a = 3$.

5.7 STANDARD IDENTITIES

Let us learn some useful identities which involve the product of two binomials.

Identity 1

$(a + b) \times (a + b)$

$$\begin{aligned}(a + b)(a + b) &= a(a + b) + b(a + b)\\ &= a \times a + a \times b + b \times a + b \times b\\ &= a^2 + ab + ba + b^2\\ &= a^2 + 2ab + b^2\end{aligned}$$

$$\mathbf{(a + b)^2 = a^2 + 2ab + b^2}$$

Identity 2

$(a - b) \times (a - b)$

$$\begin{aligned}(a - b)(a - b) &= a \times (a - b) - b \times (a - b)\\ &= a \times a + a \times (-b) + (-b) \times a + (-b) \times (-b)\\ &= a^2 - ab - ba + b^2\\ &= a^2 - 2ab + b^2\end{aligned}$$

$$\mathbf{(a - b)^2 = a^2 - 2ab + b^2}$$

Identity 3

$(a - b) \times (a + b)$

$$\begin{aligned}(a - b)(a + b) &= a \times (a + b) - b \times (a + b)\\ &= a \times a + a \times b + (-b) \times a + (-b) \times b\\ &= a^2 + ab - ba - b^2\\ &= a^2 - b^2\end{aligned}$$

$$\mathbf{(a + b)(a - b) = a^2 - b^2}$$

Identity 4

$$\begin{aligned}(x + a)(x + b) &= x \times (x + b) + a \times (a + b)\\ &= x \times x + x \times b + a \times x + a \times b\\ &= x^2 + xb + ax + ab\\ &= x^2 + (b + a)x + ab\\ &= x^2 + (a + b)x + ab\end{aligned}$$

$$\mathbf{(x + a)(x + b) = x^2 + (a + b)x + ab}$$

Let us now take up an example to find the importance to these identities.

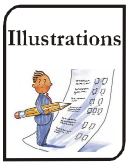


Illustration 23

Expand

$$\left(\frac{1}{2}x+\frac{3}{7}y\right)^2$$

$$\left(\frac{1}{2}x+\frac{3}{7}y\right)\left(\frac{1}{2}x+\frac{3}{7}y\right)=\frac{1}{2}\times x\left(\frac{1}{2}x+\frac{3}{7}y\right)+\frac{3}{7}y\left(\frac{1}{2}x+\frac{3}{7}y\right)$$

$$=\frac{1}{2}x\times\frac{1}{2}x+\frac{1}{2}x\times\frac{3}{7}y+\frac{3}{7}y\times\frac{1}{2}x+\frac{3}{7}y\times\frac{3}{7}y$$

$$=\frac{1}{4}x^2+\frac{3}{14}xy+\frac{3}{14}xy+\frac{9}{49}y^2$$

$$=\frac{1}{4}x^2+\left(\frac{3}{14}+\frac{3}{14}\right)xy+\frac{9}{49}y^2$$

$$=\frac{1}{4}x^2+\frac{6}{14}xy+\frac{9}{49}y^2=\frac{1}{4}x^2+\frac{3}{7}xy+\frac{9}{49}y^2$$

Let us solve this by using identity,

$$\left(\frac{1}{2}x+\frac{3}{7}y\right)^2=\left(\frac{1}{2}x\right)^2+2\times\frac{1}{2}x\times\frac{3}{7}y+\left(\frac{3}{7}y\right)^2$$

$$=\frac{1}{4}x^2+\frac{3}{7}xy+\frac{9}{49}y^2$$

Thus, we see it is much simpler to use the identity rather than multiplying the binomial by itself. If we do direct multiplication, it involves more steps and more tedious calculations. It is easier to use identities.

.

SOLVED EXAMPLE

Example : 1

Using Identity, find. (a) $(7a + 3b)^2$ (b) $(206)^2$

Solution :

(a) $(7a + 3b)^2 = (7a)^2 + 2\ (7a)\ (3b) + (3b)^2 = 49a^2 + 42ab + 9b^2$

(b) First split it as $(200 + 6)^2$

Now, $(200 + 6)^2$ can be compared to $(a + b)^2$ where a = 200, b = 6

We know that, $(a + b)^2 = a^2 + 2ab + b^2$

$\therefore$ $(200 + 6)^2 = (200)^2 + 2 \times 200 \times 6 + (6)^2 = 40000 + 2400 + 36 = 42436$

Example : 2

Using Identity, find. (a) $\left(\frac{5p}{7} - \frac{2q}{3}\right)^2$ (b) $(2p - 3q)^2$ (c) $(298)^2$

Solution :

(a) Comparing with identity, $(a - b)^2 = a^2 - 2ab + b^2$

Here, $a = \frac{5p}{7}$ and $b = \frac{2q}{3}$

$$\therefore \left(\frac{5p}{7} - \frac{2q}{3}\right)^2 = \left(\frac{5p}{7}\right)^2 - 2 \times \frac{5p}{7} \times \frac{2q}{3} + \left(\frac{2q}{3}\right)^2 = \frac{25p^2}{49} - \frac{20pq}{21} + \frac{4q^2}{9}$$

(b) $(2p - 3q)^2$ Again comparing with identity

$\therefore$ $(2p - 3q)^2 = (2p)^2 - 2 \times 2p \times 3q + (3q)^2 = 4p^2 - 12pq + 9q^2$

(c) $(298)^2$ This can be written as $(300 - 2)^2$.

Comparing with identity,

$(300 - 2)^2 = (300)^2 - 2 \times 300 \times 2 + (2)^2 = 90000 - 1200 + 4 = 88804$

Example : 3

Using identity, find (a) $(4x + 7y)\ (4x - 7y)$ (b) $\left(\frac{3a}{4} + \frac{b}{7}\right)\left(\frac{3a}{4} - \frac{b}{7}\right)$ (c) 56×64

(d) $96^2 - 4^2$

Solution :

(a) Comparing with identity, $(a + b)\ (a - b) = a^2 - b^2$

Here a = 4x and b = 7y

$\therefore$ $(4x + 7y)\ (4x - 7y) = (4x)^2 - (7y)^2 = 16x^2 - 49y^2$

(b) Comparing with identity,

$$\therefore \left(\frac{3a}{4} + \frac{b}{7}\right)\left(\frac{3a}{4} - \frac{b}{7}\right) = \left(\frac{3a}{4}\right)^2 - \left(\frac{b}{7}\right)^2 = \frac{9a^2}{16} - \frac{b^2}{49}$$

(c) This can be written as (60 – 4) (60 + 4).

Now, comparing with $(a - b)\ (a + b) = a^2 - b^2$

Here a = 60, b = 4

$56 \times 64 = (60 - 4)\ (60 + 4) = (60)^2 - (4)^2 = 3600 - 16 = 3584$

(d) Comparing with $(a + b)(a - b) = a^2 - b^2$
Hear, $a = 96$, $b = 4$
$\therefore$ $96^2 - 4^2 = (a + b)(a - b) = (96 + 4)(96 - 4) = 100 \times 92 = 9200.$

Example : 4

Using the identities evaluate :

(i) $(104)^2$ (ii) $(90)^2$ (iii) 103×97 (iv) 9.2×8.8 (v) 103×98

Solution :

(i) $(104)^2 = (100 + 4)^2 = (100)^2 + 2 \times (100 \times 4) + 4^2$ [$\because (a + b)^2 = a^2 + 2ab + b^2$]
$= 10000 + 800 + 16 = 10816$

(ii) $(90)^2 = (100 - 10)^2 = (100)^2 - 2 \times (100 \times 10) + (10)^2$ [$\because (a - b)^2 = a^2 - 2ab + b^2$]
$= 10000 - 2000 + 100 = 8100$

(iii) $103 \times 97 = (100 + 3)(100 - 3) = (100)^2 - 3^2$ [$\because (a + b)(a - b) = a^2 - b^2$]
$= 10000 - 9 = 9991$

(iv) $9.2 \times 8.8 = (9 + 0.2) \times (9 - 0.2) = 9^2 - (0.2)^2 = 81 - 0.04 = 80.96$

(v) $103 \times 98 = (100 + 3)(100 - 2) = (100)^2 + (3 - 2) \times 100 + 3 \times (-2)$
[$\because (x + a)(x + b) = x^2 + (a + b)x + ab$]
$= 10000 + 100 - 6 = 10100 - 6 = 10094$

Example : 5

If $x + \frac{1}{x} = 7$, find the value of : (i) $\left(x^2 + \frac{1}{x^2}\right)^2$ (ii) $\left(x^4 + \frac{1}{x^4}\right)$

Solution :

(i) We have, $x + \frac{1}{x} = 7$

Squaring both sides, we get

$\left(x + \frac{1}{x}\right)^2 = 7^2$ $\Rightarrow$ $x^2 + 2 \times x \times \frac{1}{x} + \frac{1}{x^2} = 49$

$\Rightarrow$ $x^2 + 2 + \frac{1}{x^2} = 49$ $\Rightarrow$ $x^2 + \frac{1}{x^2} = 49 - 2$

$\Rightarrow$ $x^2 + \frac{1}{x^2} = 47$ ....(1)

(ii) Again squaring both sides of (1) we get :

$\left(x^2 + \frac{1}{x^2}\right)^2 = (47)^2$ $\Rightarrow$ $x^4 + 2 \times x^2 \times \frac{1}{x^2} + \frac{1}{x^4} = 2209$

$\Rightarrow$ $x^4 + 2 + \frac{1}{x^4} = 2209$ $\Rightarrow$ $x^4 + \frac{1}{x^4} = 2209 - 2$

$\Rightarrow$ $x^4 + \frac{1}{x^4} = 2007$

Thus, $x^2 + \frac{1}{x^2} = 47$ and $x^4 + \frac{1}{x^4} = 2207.$

Example : 6

Find the square of 10.5 with the help of an identity.

Solution :

$(10.5)^2 = (10 + 0.5)^2 = 10^2 + 2 \times 10 \times 0.5 + (0.5)^2 = 100 + 10.0 + 0.25 = 110.25$

OR

$(10.5)^2 = (11 - 0.5)^2 = 11^2 - 2 \times 11 \times 0.5 + (0.5)^2 = 121 - 11.0 + 0.25 = 110.25$

Example : 7

Find the continued product using identities (5a –2b) (5a + 2b) (25a² – 4b²)

Solution :

$(5a - 2b)(5a + 2b)(25a^2 - 4b^2) = [(5a - 2b)(5a + 2b)](25a^2 - 4b^2)$

$= [(5a)^2 - (2b)^2](25a^2 - 4b^2)$

[Using $(x + y)(x - y) = x^2 - y^2$]

$= (25a^2 - 4b^2)(25a^2 - 4b^2)$

$= (25a^2 - 4b^2)^2$

$= (25a^2)^2 - 2 \times 25a^2 \times 4b^2 + (4b^2)^2$

[Using $(x - y)^2 = x^2 - 2xy + y^2$]

$= 625a^4 - 200a^2b^2 + 16b^4$

Example : 8

Simplify using the identities :

(i) $\dfrac{102\times102-2\times2}{102+2}$ **(ii)** $\dfrac{3.5\times3.5-2(3.5)(0.5)+0.5\times0.5}{3.5\times3.5+2(3.5)(0.5)+0.5\times0.5}$

Solution :

(i) $\dfrac{102\times102-2\times2}{102+2} = \dfrac{(102)^2-(2)^2}{104} = \dfrac{(102+2)(102-2)}{104}$ [$(a^2 - b^2) = (a + b)(a - b)$]

$= \dfrac{104\times100}{104} = 100$

(ii) $\dfrac{3.5\times3.5-2(3.5)(0.5)+0.5\times0.5}{3.5\times3.5+2(3.5)(0.5)+0.5\times0.5} = \dfrac{(3.5)^2-2(3.5)(0.5)+(0.5)^2}{(3.5)^2 2(3.5)(0.5)+(0.5)^2}$

$= \dfrac{(3.5-0.5)^2}{(3.5+0.5)^2}$

[$a^2 - 2ab + b^2 = (a - b)^2$, $a^2 + 2ab + b^2 = (a + b)^2$]

$= \dfrac{3^2}{4^2} = \dfrac{9}{16}$

Example : 9

Find the value of a if pqa = (3p + q)² – (3p – q)²

Solution :

We have $pqa = (3p + q)^2 - (3p - q)^2$

$= (3p + q + 3p - q)(3p + q - 3p + q)$ [Using $x^2 - y^2 = (x + y)(x - y)$]

$= 6p \times 2q$

$\Rightarrow$ $pqa = 12\,pq$ $\Rightarrow$ $a = \dfrac{12pq}{pq} = 12$

Example 10

Write the degree of the following algebraic expressions.

(a) $-7x^3 + 4x^2 - 3x + 9$ **(b)** $2x^4 - 4x + 11$ **(c)** $7x^5 - 4x^4 - 3x^3 + 13x - 2$

(d) $\frac{4}{3}x^3$ **(e)** 7

Solution

(a) 3 (b) 4 (c) 5 (d) 3 (e) 0

Example 11

Classify the following algebraic expressions as monomials, binomials or trinomials.

(a) $2x^5 + 7$ **(b)** $4x - 10$ **(c)** $-3x^4 + 2x^3 + 9x$ **(d)** $-2x^2 - 3x + 4$

(e) $7x^2$ **(f)** $4x^3$ **(g)** $3x^2 + 4x$ **(h)** $2x + 9y$

Solution

(a) Binomial (b) Binomial (c) Trinomial (d) Trinomial (e) Monomial (f) Monomial

(g) Binomial (h) Binomial

Example 12

Find thhe area of the rectangle whose length and breadth (in units) are :

(a) $3a^2b^3$ **and** $\frac{1}{3}ab$ **(b)** $3x^2y^5$ **and** $2x^3y$

Solution

(a) a^3b^4 sq units (b) $6x^5y^6$ sq units

Example 13

Find the volume of the rectangular tank whose length, breadth and depth (in units) are :

(a) $\frac{1}{3}xy^2, \frac{3}{5}x^2y$ **and** $\frac{2}{9}xy$ **(b)** $\frac{7}{12}xy, \frac{2}{21}xy^3, \frac{3}{5}x^3z$

Solution

(a) $\frac{2}{45}x^4y^4$ cubic units (b) $\frac{1}{30}x^5y^4z$ cubic units

Example 14

Add : 2x (z – x – y) and 2y (z – y – x).

Solution

$-4xy + 2yz + 2zx - 2x^2 - 2y^2$

CONCEPT APPLICATION LEVEL - I [NCERT Questions]

EXERCISE - 1

Q.1 Identify the terms, their coefficients for each of the following expressions:

(i) $5xyz^2 - 3zy$ **(ii)** $1 + x + x^2$ **(iii)** $4x^2y^2 - 4x^2y^2z^2 + z^2$

(iv) $3 - pq + qr - rp$ **(v)** $\frac{x}{2} + \frac{y}{2} - xy$ **(vi)** $0.3a - 0.6ab + 0.5b$

Sol.

(i) $5xyz^2 - 3zy$: Terms : $5xyz^2$, $-3zy$
Their coefficients : 5, -3

(ii) $1 + x + x^2$: Terms : 1, x, x^2
Their coefficients : 1, 1, 1

(iii) $4x^2y^2 - 4x^2y^2z^2 + z^2$ Terms : $4x^2y^2$, $-4x^2y^2z^2$, z^2
Their coefficients : 4, -4, 1

(iv) $3 - pq + qr - rp$ Terms : 3, $-pq$, qr, $-rp$
Their coefficients : 3, -1, 1 -1

(v) $\frac{x}{2} + \frac{y}{2} - xy$ Terms : $\frac{x}{2}$, $\frac{y}{2}$, $-xy$
Their coefficients : $\frac{1}{2}$, $\frac{1}{2}$, -1

(vi) $0.3a - 0.6ab + 0.5b$ Terms : $0.3a$, $-0.6ab$, $0.5b$
Their coefficients : 0.3, -0.6, 0.5

Q.2 Classify the following polynomials as monomials, binomials, trinomials. Which polynomials do not fit in any of these three categories?
$x + y$, 1000, $x + x^2 + x^3 + x^4$, $7 + y + 5x$, $2y - 3y^2$, $2y - 3y^2 + 4y^3$, $5x - 4y + 3xy$, $4z - 15z^2$, $ab + bc + cd + da$, pqr, $p^2q + pq^2$, $2p + 2q$.

Sol.

Monomials	**Binomials**	**Trinomials**	**Polynomials that do not fit in these categories**
1000 pqr	$x + y$ $2y - 3y^2$ $4z - 15z^2$ $p^2q + pq^2$ $2p + 2q$	$7 + y + 5x$ $2y^2 - 3y^2 + 4y^3$ $5x - 4y + 3xy$	$x + x^2 + x^3 + x^4$ $ab + bc + cd + da$

Q.3 Add the following.

(i) $ab - bc, bc - ca, ca - ab$

(ii) $a - b + ab, b - c + bc, c - a + ac$

(iii) $2p^2q^2 - 3pq + 4, 5 + 7pq - 3p^2q^2$

(iv) $l^2 + m^2, m^2 + n^2, n^2 + l^2, 2lm + 2mn + 2nl$

Sol. (i) $ab - bc, bc - ca, ca - ab$

$$\begin{array}{rrrr} & ab & -bc & \\ + & & +bc & -ca \\ + & -ab & & ca \\ \hline & & 0 & \\ \hline \end{array}$$

(ii) $a - b + ab, b - c + bc, c - a + ac$

$$\begin{array}{rrrrrr} & a & -b & +ab & & \\ + & & +b & & -c & +bc \\ + & -a & & & +c & +ac \\ \hline & & & ab & + bc & +ac \\ \hline \end{array}$$

(iii) $2p^2q^2 - 3pq + 4, 5 + 7pq - 3p^2q^2$

$$\begin{array}{rrrr} & 2p^2q^2 & -3pq & +4 \\ + & -3p^2q^2 & +7pq & +5 \\ \hline & -p^2q^2 & +4pq & +9 \\ \hline \end{array}$$

(iv) $l^2 + m^2, m^2 + n^2, n^2 + l^2, 2lm + 2mn + 2nl$

$$\begin{array}{rrrrl} & l^2 & +m^2 & & \\ + & & +m^2 & +n^2 & \\ + & l^2 & & +n^2 & \\ + & & & & +2lm + 2mn + 2nl \\ \hline \end{array}$$

$2l^2 + 2m^2 + 2n^2 + 2lm + 2mn + 2nl$

$= 2(l^2 + m^2 + n^2 + lm + mn + nl)$

Q.4 (a) Subtract $4a - 7ab + 3b + 12$ from $12a - 9ab + 5b - 3$.

(b) Subtract $3xy + 5yz - 7zx$ from $5xy - 2yz - 2zx + 10xyz$

(c) Subtract $4p^2q - 3pq + 5pq^2 - 8p + 7q - 10$ from $18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q$.

Sol. (a)

$$\begin{array}{rrrrr} & 12a & -9ab & +5b & -3 \\ - & 4a & -7ab & +3b & +12 \\ & - & + & - & - \\ \hline & 8a & -2ab & +2b & -15 \\ \hline \end{array}$$

(b) $5xy - 2yz - 2zx + 10xyz$
$-$ $3xy + 5yz - 7zx$
$\quad - \quad - \quad +$

$2xy - 7yz + 5zx + 10xyz$

(c) $18 - 3p - 11q + 5pq - 2pq^2 + 5p^2q$
$-$ $-10 - 8p + 7q - 3pq + 5pq^2 + 4p^2q$
$\quad + \quad + \quad - \quad + \quad - \quad -$

$28 + 5p - 18q + 8pq - 7pq^2 + p^2q$

EXERCISE-2

Q.1 Find the product of the following pairs of monomials :

(i) 4, 7p (ii) –4p, 7p (iii) –4p, 7pq (iv) $4p^3$, –3p (v) 4p, 0.

Sol. (i) 4, 7p

$4 \times 7p = (4 \times 7) \times p$
$= 28 \times p = 28p$

(ii) –4p, 7p

$(-4p) \times (7p) = \{(-4) \times 7\} \times (p \times p)$
$= (-28) \times p^2 = -28p^2$

(iii) –4p, 7pq

$(-4p) \times (7pq) = \{(-4) \times 7\} \times \{p \times (pq)\}$
$= (-28) \times (p \times p \times p)$
$= (-28) \times (p^2q) = -28\,p^2q$

(iv) $4p^3$, –3p

$(4p^3) \times (-3p) = \{4 \times (-3)\} \times (p^3 \times p)$
$= (-12) \times p^4 = -12p^4$

(v) 4p, 0

$(4p) \times 0 = (4 \times 0) \times p$
$= 0 \times p = 0$

Q.2 Find the areas of rectangles with the following pairs of monomials as their lengths and breadths respectively :

(i) (p, q) (ii) (10m, 5n) (iii) $(20x^2, 5y^2)$ (iv) $(4x, 3x^2)$ (v) (3mn, 4np)

Sol. (i) (p, q) : Area of the rectangle = Length × Breadth
$= p \times q = pq$

(ii) (10m, 5n) Area of the rectangle = Length × Breadth
$= (10m) \times (5n)$
$= (10 \times 5) \times (m \times n)$
$= 50 \times (mn) = 50\,mn$

(iii) **($20x^2, 5y^2$)** Area of the rectangle = Length × Breadth

$= (20x^2) \times (5y^2)$

$= (20 \times 5) \times (x^2 \times y^2)$

$= 100 \times (x^2y^2) = 100x^2y^2$

(iv) **($4x, 3x^2$)** Area of the rectangle = Length × Breadth

$= (4x) \times (3x^2)$

$= (4 \times 3) \times (x \times x^2)$

$= 12 \times x^3 = 12x^3$

(v) **(3mn, 4np)** Area of the rectangle = Length × Breadth

$= (3mn) \times (4np)$

$= (3 \times 4) \times (mn) \times (np)$

$= 12 \times m \times (n \times n) \times p$

$= 12 \times mn^2p$

Q.3 Complete the table of products

First monomial → / **Second monomial ↓**	**$2x$**	**$-5y$**	**$3x^2$**	**$-4xy$**	**$7x^2y$**	**$-9x^2y^2$**
$2x$	**$4x^2$**	—	—	—	—	—
$-5y$	—	—	**$-15x^2y$**	—	—	—
$3x^2$	—	—	—	—	—	—
$-4xy$	—	—	—	—	—	—
$7x^2y$	—	—	—	—	—	—
$-9x^2y^2$	—	—	—	—	—	—

Sol.

First monomial → / **Second monomial ↓**	**$2x$**	**$-5y$**	**$3x^2$**	**$-4xy$**	**$7x^2y$**	**$-9x^2y^2$**
$2x$	$4x^2$	$-10xy$	$6x^3$	$-8x^2y$	$14x^3y$	$-18x^3y^2$
$-5y$	$-10xy$	$25y^2$	$-15x^2y$	$20xy^2$	$-35x^2y^2$	$45x^2y^3$
$3x^2$	$6x^3$	$-15x^2y$	$9x^4$	$-12x^3y$	$21x^4y$	$-27x^4y^2$
$-4xy$	$-8x^2y$	$20xy^2$	$-12x^3y$	$16x^2y^2$	$-28x^3y^2$	$36x^3y^3$
$7x^2y$	$14x^3y$	$-35x^2y^2$	$21x^4y$	$-28x^3y^2$	$49x^4y^2$	$-63x^4y^3$
$-9x^2y^2$	$-18x^3y^2$	$45x^2y^3$	$-27x^4y^2$	$36x^3y^3$	$-63x^4y^3$	$81x^4y^4$

Q.4 Obtain the value of rectangular boxes with the following length, breadth and height respectively:
(i) $5a, 3a^2, 7a^4$ (ii) 2p, 4q, 8r (iii) $xy, 2x^2y, 2xy^2$ (iv) a, 2b, 3c

Sol. (i) $5a, 3a^2, 7a^4$ Volume of the rectangular box = Length × Breadth × Height
$= (5a) \times (3a^2) \times (7a^4)$
$= (5 \times 3 \times 7) \times (a \times a^2 \times a^4)$
$= 105a^7$

(ii) 2p, 4q, 8r Volume of the rectangular box = Length × Breadth × Height
$= (2p) \times (4q) \times (8r)$
$= (2 \times 4 \times 8) \times (p \times q \times r)$
$= 64pqr$

(iii) $xy, 2x^2y, 2xy^2$ Volume of the rectangular box = Length × Breadth × Height
$= (xy) \times (2x^2y) \times (2xy^2)$
$= (2 \times 2) \times (x \times x^2 \times x) \times (y \times y \times y^2)$
$= 4x^4y^4$

(iv) a, 2b, 3c Volume of the rectangular box = Length × Breadth × Height
$= (a) \times (2b) \times (3c)$
$= (2 \times 3) \times (a \times b \times c)$
$= 6abc$

Q.5 Obtain the product of
(i) xy, yz, zx (ii) $a, -a^2, a^3$ (iii) $2, 4y, 8y^2, 16y^3$ (iv) a, 2b, 3c, 6abc
(v) m, –mn, mnp

Sol. (i) xy, yz, zx Required product $= (xy) \times (yz) \times (zx)$
$= (x \times x) \times (y \times y) \times (z \times z)$
$= x^2 \times y^2 \times z^2 = x^2y^2z^2$

(ii) $a, -a^2, a^3$ Required product $= (a) \times (-a^2) \times (a^3)$
$= -(a \times a^2 \times a^3) = -a^6$

(iii) $2, 4y, 8y^2, 16y^3$ Required product $= (2) \times (4y) \times (8y^2) \times (16y^3)$
$= (2 \times 4 \times 8 \times 16) \times (y \times y^2 \times y^3) = 1024y^6$

(iv) a, 2b, 3c, 6abc Required product $= (a) \times (2b) \times (3c) \times (6abc)$
$= (2 \times 3 \times 6) \times (a \times a) \times (b \times b) \times (c \times c)$
$= 36a^2b^2c^2$

(v) m, –mn, mnp Required product $= (m) \times (-mn) \times (mnp)$
$= (-1) \times (m \times m \times m) \times (n \times n) \times p$
$= -m^3 n^2 p$

EXERCISE - 3

Q.1 Carry out the multiplication of the expression in each of the following pairs :

(i) $4p, q + r$ (ii) $ab, a - b$ (iii) $a + b, 7a^2b^2$ (iv) $a^2 - 9, 4a$

(v) $pq + qr + 2p, 0$

Sol. (i) $4p, q + r$

$(4p) \times (q + r) = (4p) \times (q) + (4p) \times (r)$

$= 4pq + 4pr$

(ii) $ab, a - b$

$(ab) \times (a - b) = (ab) \times (a) - (ab) \times (b)$

$= a^2b - ab^2$

(iii) $a + b, 7a^2b^2$

$(a + b) \times (7a^2b^2) = (7a^2b^2) \times (a + b)$ (by commutative law)

$= (7a^2b^2) \times (a) + (7a^2b^2) \times (b)$

$= 7a^3b^2 + 7a^2b^3$

(iv) $a^2 - 9, 4a$

$(a^2 - 9) \times (4a) = (4a) \times (a^2 - 9)$ (by commutative law)

$= (4a) \times (a^2) - (4a) \times (9)$

$= 4a^3 - 36a$

(v) $pq + qr + 2p, 0$

$(pq + qr + 2p) \times (0) = (0) \times (pq + pr + 2p)$ (by commutative law)

$= (0) \times (pq) + (0) \times (qr) + (0) \times (2p)$

$= 0 + 0 + 0 = 0$

Q.2 Complete the table :

	First expression	**Second expression**	**Product**
(i)	**a**	**$b + c + d$**	—
(ii)	**$x + y - 5$**	**$5xy$**	—
(iii)	**p**	**$6p^2 - 7p + 5$**	—
(iv)	**$4p^2q^2$**	**$p^2 - q^2$**	—
(v)	**$a + b + c$**	**abc**	—

Sol.

	First expression	**Second expression**	**Product**
(i)	a	$b + c + d$	$ab + ac + ad$
(ii)	$x + y - 5$	$5xy$	$5x^2y + 5xy^2 - 25xy$
(iii)	p	$6p^2 - 7p + 5$	$6p^3 - 7p^2 + 5p$
(iv)	$4p^2q^2$	$p^2 - q^2$	$4p^4q^2 - 4p^2q^4$
(v)	$a + b + c$	abc	$a^2bc + ab^2c + abc^2$

Q.3 Find the product :

(i) $(a^2) \times (2a^{22}) \times (4a^{26})$ **(ii)** $\left(\frac{2}{3}xy\right) \times \left(\frac{-9}{10}x^2y^2\right)$

(iii) $\left(-\frac{10}{3}pq^3\right) \times \left(\frac{6}{5}p^3q\right)$ **(iv)** $x \times x^2 \times x^3 \times x^4$

Sol. (i) $(a^2) \times (2a^{22}) \times (4a^{26}) = (2 \times 4) \times (a^2 \times a^{22} \times a^{26}) = 8 \times a^{50} = 8a^{50}$.

(ii) $\left(\frac{2}{3}xy\right) \times \left(\frac{-9}{10}x^2y^2\right) = \left\{\frac{2}{3} \times \left(-\frac{9}{10}\right)\right\} \times (x \times x^2) \times (y \times y^2) = -\frac{3}{5}x^3y^3$.

(iii) $\left(-\frac{10}{3}pq^3\right) \times \left(\frac{6}{5}p^3q\right) = \left\{\left(-\frac{10}{3}\right) \times \frac{6}{5}\right\} \times (p \times p^3) \times (q^3 \times q) = -4p^4q^4$

(iv) $x \times x^2 \times x^3 \times x^4 = x \times x^2 \times x^3 \times x^4 = \times x^1 \times x^2 \times x^3 \times x^4 = x^{1+2+3+4} = x^{10}$

Q.4 (a) Simplify : 3x (4x – 5) + 3 and find its values for (i) x = 3, (ii) x = $\frac{1}{2}$.

(b) Simplify : $a(a^2 + a + 1) + 5$ and find its value for (i) a = 0, (ii) a = 1 and (iii) a = –1.

Sol. (a) $3x (4x - 5) + 3 = (3x) (4x) - (3x) (5) + 3$
$= (3 \times 4) \times (x \times x) - 15x + 3$
$= 12x^2 - 15x + 3$

(i) When x = 3,
$12x^2 - 15x + 3 = 12(3)^2 - 15(3) + 3 = 108 - 45 + 3 = 66$

(ii) When x = $\frac{1}{2}$,

$$12x^2 - 15x + 3 = 12\left(\frac{1}{2}\right)^2 - 15\left(\frac{1}{2}\right) + 3 = 3 - \frac{15}{2} + 3 = -\frac{3}{2}$$

(b) $a(a^2 + a + 1) + 5 = a \times a^2 + a \times a + a \times 1 + 5 = a^3 + a^2 + a + 5$

(i) When a = 0
$a^3 + a^2 + a + 5 = 0 = (0)^3 + (0)^2 + (0) + 5$

(ii) When a = 1
$a^3 + a^2 + a + 5 = (1)^3 + (1)^2 + (1) + 5 = 1 + 1 + 1 + 5 = 8$

(iii) When a = –1
$a^3 + a^2 + a + 5 = (-1)^3 + (-1)^2 + (-1) + 5 = -1 + 1 - 1 + 5 = 4$

Q.5 **(a) Add : p(p – q), q(q – r) and r(r – p)**
(b) Add : 2x(z – x – y) and 2y(z – y – x)
(c) Subtract : $3l(l - 4m + 5n)$ from $4l(10n - 3m + 2l)$
(d) Subtract : 3a(a + b + c) – 2b(a – b + c) from 4c (–a + b + c)

Sol. **(a)** First expression $= p(p - q) = p \times p - p \times q = p^2 - pq$
Second expression $= q(q - r) = q \times q - q \times r = q^2 - qr$
Third expression $= r(r - p) = r \times r - r \times p = r^2 - rp$
Adding the three expressions,

$$\begin{array}{lccc} & p^2 - pq & & \\ + & & + q^2 - qr & \\ + & & & + r^2 - rp \\ \hline & \multicolumn{3}{c}{p^2 - pq + q^2 - qr + r^2 - rp} \\ \hline \end{array}$$

(b) First expression $= 2x(z - x - y) = (2x) \times (z) - (2x) \times (x) - (2x) \times (y) = 2xz - 2x^2 - 2xy$
Second expression $= 2y(z - y - x) = (2y) \times (z) - (2y) \times (y) - (2y) \times (x) = 2yz - 2y^2 - 2yx$
Adding the two expressions,

$$\begin{array}{lcc} & 2xz - 2x^2 - 2xy & \\ + & & - 2yx + 2yz - 2y^2 \\ \hline & \multicolumn{2}{c}{2xz - 2x^2 - 4xy + 2yz - 2y^2} \\ \hline \end{array}$$

(c) First expression $= 3l\,(l - 4m + 5n) = (3l) \times (l) - (3l) \times (4m) + (3l) \times (5n) = 3l^2 - 12lm + 15ln$
Second expression $= 4l\,(10n - 3m + 2l) = (4l) \times (10n) - (4l) \times (3m) + (4l) \times (2l)$
$= 40ln - 12lm + 8l^2$
Subtraction ,

$$\begin{array}{l} 40ln - 12lm + 8l^2 \\ 15ln - 12lm + 3l^2 \\ - \quad\; + \qquad - \\ \hline 25ln \qquad\quad + 5l^2 \\ \hline \end{array}$$

(d) First expression $= 3a\,(a + b + c) - 2b\,(a - b + c)$
$= (3a) \times (a) + (3a) \times (b) + (3a) \times (c) - (2b) \times (a) + (2b) \times (b) - (2b) \times (c)$
$= 3a^2 + 3ab + 3ac - 2ab + 2b^2 - 2bc$
$= 3a^2 + 2b^2 + 3ab - 2ab - 2bc + 3ac$
$= 3a^2 + 2b^2 + ab - 2bc + 3ac$
Second expression
$= 4c\,(-a + b + c) = 4c \times (-a) + 4c \times b + 4c \times c$
$= -4ac + 4bc + 4c^2$
Subtraction ,

$$\begin{array}{l} \qquad\qquad\quad -4ac + 4bc + 4c^2 \\ 3a^2 + 2b^2 + ab + 3ac - 2bc \\ - \quad - \quad - \quad - \quad + \\ \hline -3a^2 - 2b^2 - ab - 7ac + 6bc + 4c^2 \\ \hline \end{array}$$

EXERCISE - 4

Q.1 Multiply the binomials :

(i) $(2x + 5)$ and $(4x - 3)$ **(ii)** $(y - 8)$ and $(3y - 4)$

(iii) $(2.5l - 0.5\,m)$ and $(2.5\,l + 0.5\,m)$ **(iv)** $(a + 3b)$ and $(x + 5)$

(v) $(2pq + 3q^2)$ and $(3pq - 2q^2)$ **(vi)** $\left(\frac{3}{4}a^2 + 3b^2\right)$ and $4\left(a^2 - \frac{2}{3}b^2\right)$

Sol. (i) $(2x + 5)$ and $(3y - 4)$

$(2x + 5) \times (4x - 3) = (2x) \times (4x - 3) + 5 \times (4x - 3)$

$= (2x) \times (4x) - (2x) \times (3) + (5) \times (4x) - (5) \times (3)$

$= 8x^2 - 6x + 20x - 15$

$= 8x^2 + (20x - 6x) - 15$ (Combining like terms)

$= 8x^2 + 14x - 15$

(ii) $(y - 8)$ and $(4x - 3)$

$(y - 8) \times (3y - 4) = y \times (3y - 4) - 8 \times (3y - 4)$

$= (y) \times (3y) - (y) \times (4) - (8) \times (3y) + 8 \times 4$

$= 3y^2 - 4y - 24y + 32$

$= 3y^2 - 28y + 32$ (Combining like terms)

(iii) $(2.5l - 0.5\,m)$ and $(2.5\,l + 0.5\,m)$

$(2.5l - 0.5\,m) \times (2.5\,l + 0.5\,m)$

$= (2.5l) \times (2.5l + 0.5m) - (0.5m) \times (2.5l + 0.5m)$

$= (2.5l) \times (2.5l) + (2.5l) \times (0.5m) - (0.5\,m) \times (2.5l) - (0.5m) \times (0.5m)$

$= 6.25l^2 + 1.25lm - 1.25ml - 0.25m^2$

$= 6.25l^2 + (1.25lm - 1.25ml) - 0.25m^2$ (Combining like terms)

$= 6.25l^2 - 0.25m^2$

(iv) $(a + 3b)$ and $(x + 5)$

$(a + 3b) \times (x + 5) = a \times (x + 5) + (3b) \times (x + 5)$

$= (a) \times (x) + (a) \times (5) + (3b) \times (x) + (3b) \times (5)$

$= ax + 5a + 3bx + 15b$

(v) $(2pq + 3q^2)$ and $(3pq - 2q^2)$

$(2pq + 3q^2) \times (3pq - 2q^2)$

$= (2pq) \times (3pq - 2q^2) + (3q^2) \times (3pq - 2q^2)$

$= (2pq) \times (3pq) - (2pq) \times (2q^2) + (3q^2) \times (3pq) - (3q^2) \times (2q^2)$

$= 6p^2q^2 - 4pq^3 + 9pq^3 - 6q^4$

$= 6p^2q^2 - (9pq^3 - 4pq^3) - 6q^4$ (Combining like terms)

$= 6p^2q^2 + 5pq^3 - 6q^4$

(vi) $\left(\frac{3}{4}a^2 + 3b^2\right)$ **and** $4\left(a^2 - \frac{2}{3}b^2\right)$

$\left(\frac{3}{4}a^2 + 3b^2\right) \times 4\left(a^2 - \frac{2}{3}b^2\right)$

$= \left(\frac{3}{4}a^2 + 3b^2\right) \times \left(4a^2 - \frac{8}{3}b^2\right)$

$= \frac{3}{4}a^2 \times \left(4a^2 - \frac{8}{3}b^2\right) + 3b^2 \times \left(4a^2 - \frac{8}{3}b^2\right)$

$= \left(\frac{3}{4}a^2\right) \times (4a^2) - \left(\frac{3}{4}a^2\right) \times \left(\frac{8}{3}b^2\right) + 3b^2 \times (4a^2) - (3b^2) \times \left(\frac{8}{3}b^2\right)$

$= 3a^4 - 2a^2b^2 + 12b^2a^2 - 8b^4$

$= 3a^4 + (12a^2b^2 - 2a^2b^2) - 8b^4$ (Combining like terms)

$= 3a^4 + 10a^2b^2 - 8b^4$

Q.2 Find the product :

(i) (5 – 2x) (3 + x) **(ii) (x + 7y) (7x – y)** **(iii) $(a^2 + b)(a + b^2)$**

(iv) $(p^2 - q^2)(2p + q)$

Sol. (i) (5 – 2x) (3 + x)

$(5 - 2x) \times (3 + x) = (5) \times (3 + x) - (2x) \times (3 + x)$

$= (5) \times (3) + (5) \times (x) - (2x) \times (3) - (2x) \times (x)$

$= 15 + 5x - 6x - 2x^2$

$= 15 - x - 2x^2$ (Combining like terms)

(ii) (x + 7y) (7x – y)

$(x + 7y) \times (7x - y) = (x) \times (7x - y) + (7y) \times (7x - y)$

$= (x) \times (7x) - (x) \times (y) + (7y) \times (7x) - (7y) \times (y)$

$= 7x^2 - xy + 49yx - 7y^2$

$= 7x^2 + 48xy - 7y^2$ (Combining like terms)

(iii) $(a^2 + b)(a + b^2)$

$(a^2 + b) \times (a + b^2) = a^2 \times (a + b^2) + b \times (a + b^2)$

$= (a^2) \times (a) + (a^2) \times (b^2) + (b) \times (a) + (b) \times (b^2)$

$= a^3 + a^2b^2 + ba + b^3$

(iv) $(p^2 - q^2)(2p + q)$

$(p^2 - q^2) \times (2p + q) = p^2 \times (2p + q) - q^2 \times (2p + q)$

$= (p^2) \times (2p) + (p^2) \times (q) - (q^2) \times (2p) - (q^2) \times (q)$

$= 2p^3 + p^2q - 2q^2p - q^3$

Q.3 Simplify :

(i) $(x^2 - 5)(x + 5) + 25$ **(ii)** $(a^2 + 5)(b^3 + 3) + 5$

(iii) $(t + s^2)(t^2 - s)$ **(iv)** $(a + b)(c - d) + (a - b)(c + d) + 2(ac + bd)$

(v) $(x + y)(2x + y) + (x + 2y)(x - y)$ **(vi)** $(x + y)(x^2 - xy + y^2)$

(vii) $(1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y$ **(viii)** $(a + b + c)(a + b - c)$

Sol. (i) $(x^2 - 5)(x + 5) + 25$
$= x^2(x + 5) - 5(x + 5) + 25$
$= x^3 + 5x^2 - 5x - 25 + 25$
$= x^3 + 5x^2 - 5x$

(ii) $(a^2 + 5)(b^3 + 3) + 5$
$= a^2(b^3 + 3) + 5(b^3 + 3) + 5$
$= a^2b^3 + 3a^2 + 5b^3 + 15 + 5$
$= a^2b^3 + 3a^2 + 5b^3 + 20$

(iii) $(t + s^2)(t^2 - s)$
$= t(t^2 - s) + s^2(t^2 - s)$
$= t^3 - ts + s^2t^2 - s^3$

(iv) $(a + b)(c - d) + (a - b)(c + d) + 2(ac + bd)$
$= a(c - d) + b(c - d) + a(c + d) - b(c + d) + 2(ac + bd)$
$= ac - ad + bc - bd + ac + ad - bc - bd + 2ac + 2bd$
$= (ac + ac + 2ac) + (ad - ad) + (bc - bc) + (2bd - bd - bd)$
(Combining like terms)

(v) $(x + y)(2x + y) + (x + 2y)(x - y)$
$= x(2x + y) + y(2x + y) + x(x - y) + 2y(x - y)$
$= 2x^2 + xy + 2xy + y^2 + x^2 - xy + 2xy - 2y^2$
$= (2x^2 + x^2) + (xy + 2xy - xy + 2xy) + (y^2 - 2y^2)$
(Combining like terms)
$= 3x^2 + 4xy - y^2$

(vi) $(x + y)(x^2 - xy + y^2)$
$= x(x^2 - xy + y^2) + y(x^2 - xy + y^2)$
$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$
$= x^3 + (x^2y - x^2y) + (xy^2 - xy^2) + y^3$ (Combining like terms)
$= x^3 + y^3$

(vii) $(1.5x - 4y)(1.5x + 4y + 3) - 4.5x + 12y$
$= 1.5x(1.5x + 4y + 3) - 4y(1.5x + 4y + 3) - 4.5x + 12y$
$= 2.25x^2 + 6xy + 4.5x - 6xy - 16y^2 - 12y - 4.5x + 12y$
$= 2.25x^2 + (6xy - 6xy) - 16y^2 + (4.5x - 4.5x) + (12y - 12y)$
(Combining like terms)
$= 2.25x^2 - 16y^2$

(viii) $(a + b + c)(a + b - c)$
$= a(a + b - c) + b(a + b - c) + c(a + b - c)$
$= a^2 + ab - ac + ab + b^2 - bc + ac + bc - c^2$
$= a^2 + (ab + ab) + (ac - ac) + b^2 + (bc - bc) - c^2$
(Combining like terms)
$= a^2 + 2ab + b^2 - c^2$

EXERCISE - 5

Q.1 Use a suitable identity to get each of the following products :

(i) $(x + 3)(x + 3)$ (ii) $(2y + 5)(2y + 5)$ (iii) $(2a - 7)(2a - 7)$

(iv) $\left(3a - \frac{1}{2}\right)\left(3a - \frac{1}{2}\right)$ (v) $(1.1m - 0.4)(1.1m + 0.4)$ (vi) $(a^2 + b^2)(-a^2 + b^2)$

(vii) $(6x - 7)(6x + 7)$ (viii) $(-a + c)(-a + c)$ (ix) $\left(\frac{x}{2} + \frac{3y}{4}\right)\left(\frac{x}{2} + \frac{3y}{4}\right)$

(x) $(7a - 9b)(7a - 9b)$

Sol. (i) $(x + 3)(x + 3) = (x + 3)^2$

$= (x)^2 + 2(x)(3) + (3)^2$ (using Identity I)

$= x^2 + 6x + 9$

(ii) $(2y + 5)(2y + 5) = (2y + 5)^2$

$= (2y)^2 + 2(2y)(5) + (5)^2$ (using Identity I)

$= 4y^2 + 20y + 25$

(iii) $(2a - 7)(2a - 7) = (2a - 7)^2$

$= (2a)^2 - 2(2a)(7) + (7)^2$

$= 4a^2 - 28a + 49$

(iv) $\left(3a - \frac{1}{2}\right)\left(3a - \frac{1}{2}\right) = \left(3a - \frac{1}{2}\right)^2$

$= (3a)^2 - 2(3a)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^2$ (Using Identity II)

$= 9a^2 - 3a + \frac{1}{4}$

(v) $(1.1m - 0.4)(1.1m + 0.4) = (1.1m)^2 - (0.4)^2$ (Using Identity III)

$= 1.21m^2 - 0.16$

(vi) $(a^2 + b^2)(-a^2 + b^2) = (b^2 + a^2)(b^2 - a^2)$

$= (b^2)^2 - (a^2)^2$ (Using Identity III)

$= b^4 - a^4$

(vii) $(6x - 7)(6x + 7) = (6x)^2 - (7)^2$ (Using Identity III)

$= 36x^2 - 49$

(viii) $(-a + c)(-a + c) = (-a + c)^2$

$= (c - a)^2$

$= c^2 - 2ca + a^2$ (Using Identity II)

(ix) $\left(\frac{x}{2}+\frac{3y}{4}\right)\left(\frac{x}{2}+\frac{3y}{4}\right)=\left(\frac{x}{2}\right)^2+2\left(\frac{x}{2}\right)\left(\frac{3y}{4}\right)+\left(\frac{3y}{4}\right)^2$ (Using Identity I)

$=\frac{x^2}{4}+\frac{3xy}{4}+\frac{9y^2}{16}$

(x) **(7a – 9b) (7a – 9b)** $= (7a-9b)^2$

$= (7a)^2 - 2(7a)(9b) + (9b)^2$ (Using Identity II)

$= 49a^2 - 126ab + 81ab^2$

Q.2 Use the identity $(x + a)(x + b) = x^2 + (a + b)x + ab$ to find the following products :

(i) (x + 3) (x + 7) **(ii) (4x + 5) (4x + 1)** **(iii) (4x – 5) (4x – 1)**

(iv) (4x + 5) (4x – 1) **(v) (2x + 5y) (2x + 3y)** **(vi) $(2a^2 + 9)(2a^2 + 5)$**

(viii) (xyz – 4) (xyz – 2)

Sol. **(i)** **(x + 3) (x + 7)** $= x^2 + (3 + 7)x + (3)(7)$

$= x^2 + 10x + 21$

(ii) **(4x + 5) (4x + 1)** $= (4x)^2 + (5 + 1)(4x) + (5)(1)$

$= 16x^2 + 24x + 5$

(iii) **(4x – 5) (4x – 1)** $= \{4x + (-5)\}\{4x + (-1)\}$

$= (4x)^2 + \{(-5) + (-1)\}(4x) + (-5)(-1)$

$= 16x^2 - 24x + 5$

(iv) **(4x + 5) (4x – 1)** $= (4x + 5)\{4x + (-1)\}$

$= (4x)^2 + \{5 + (-1)\}(4x) + (5)(-1)$

$= 16x^2 + 16x - 5$

(v) **(2x + 5y) (2x + 3y)** $= (2x)^2 + (5y + 3y)(2x) + (5y)(3y)$

$= 4x^2 + (8y)(2x) + 15y^2$

$= 4x^2 + 16xy + 15y^2$

(vi) **$(2a^2 + 9)(2a^2 + 5)$** $= (2a^2)^2 + (5 + 9)(2a)^2 + (5)(9)$

$= 4a^4 + 28a^2 + 45$

(vii) **(xyz – 4) (xyz – 2)** $= \{xyz + (-4)\}\{xyz + (-2)\}$

$= (xyz)^2 + \{(-4) + (-2)\}(xyz) + (-4)(-2)$

$= x^2y^2z^2 - 6xyz + 8.$

Q.3 Find the following squares by using the identities.

(i) $(b-7)^2$ **(ii) $(xy+3z)^2$** **(iii) $(6x^2-5y)^2$** **(iv) $\left(\frac{2}{3}m+\frac{3}{2}n\right)^2$**

(v) $(0.4p-0.5q)^2$ **(vi) $(2xy+5y)^2$.**

Sol. (i) $(b-7)^2 = (b-7)(b-7)$

$= b(b-7) - 7(b-7)$

$= b^2 - 7b - 7b + 49$

$= b^2 - 14b + 49$

(ii) $(xy+3z)^2 = (xy+3z)(xy+3z)$

$= xy(xy+3z) + 3z(xy+3z)$

$= x^2y^2 + 3xyz + 3xyz + 9z^2$

$= x^2y^2 + 6xyz + 9z^2$

(iii) $(6x^2-5y)^2 = (6x^2-5y)(6x^2-5y)$

$= 6x^2(6x^2-5y) - 5y(6x^2-5y)$

$= 36x^4 - 30x^2y - 30x^2y + 25y^2$

$= 36x^4 - 60x^2y + 25y^2$

(iv) $\left(\frac{2}{3}m+\frac{3}{2}n\right)^2 = \left(\frac{2}{3}m+\frac{3}{2}n\right)\left(\frac{2}{3}m+\frac{3}{2}n\right)$

$$= \frac{2}{3}m\left(\frac{2}{3}m+\frac{3}{2}n\right) + \frac{3}{2}n\left(\frac{2}{3}m+\frac{3}{2}n\right)$$

$$= \frac{4}{9}m^4 + mn + mn + \frac{9}{4}n^2$$

$$= \frac{4}{9}m^2 + 2mn + \frac{9}{4}n^2$$

(v) $(0.4p-0.5q)^2 = (0.4p-0.5q)(0.4p-0.5q)$

$= 0.4p(0.4p-0.5q) - 0.5q(0.4p-0.5q)$

$= 0.16p^2 - 0.2pq - 0.2pq + 0.25q^2$

$= 0.16p^2 - 0.4pq + 0.25q^2$

(vi) $(2xy+5y)^2 = (2xy+5y)(2xy+5y)$

$= 2xy(2xy+5y) + 5y(2xy+5y)$

$= 4x^2y^2 + 10xy^2 + 10xy^2 + 25y^2$

$= 4x^2y^2 + 20xy^2 + 25y^2$

Q.4 Simplify :

(i) $(a^2 - b^2)^2$ **(ii) $(2x + 5)^2 - (2x - 5)^2$**

(iii) $(7m - 8n)^2 + (7m + 8n)^2$ **(iv) $(4m + 5n)^2 + (5m + 4n)^2$**

(v) $(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$ **(vi) $(ab + bc)^2 - 2ab^2c$**

(vii) $(m^2 - n^2m)^2 + 2m^3n^2$

Sol. (i) $(a^2 - b^2)^2 = (a^2)^2 - 2(a^2)(b^2) + (b^2)^2$
$= a^4 - 2a^2b^2 + b^4$

(ii) $(2x + 5)^2 - (2x - 5)^2 = \{(2x)^2 + 2(2x)(5) + (5)^2\} - \{(2x)^2 - 2(2x)(5) + (5)^2\}$
$= (4x^2 + 20x + 25) - (4x^2 - 20x + 25)$
$= 4x^2 + 20x + 25 - 4x^2 + 20x - 25 = 40x$

(iii) $(7m - 8n)^2 + (7m + 8n)^2 = \{(7m)^2 - 2(7m)(8n) + (8n)^2\} + \{(7m)^2 + 2(7m)(8n) + (8n)^2\}$
$= (49m^2 - 112mn + 64n^2) + (49m^2 + 112mn + 64n^2)$
$= 2(49m^2 + 64n^2) = 98m^2 + 128n^2$

(iv) $(4m + 5n)^2 + (5m + 4n)^2 = \{(4m)^2 + 2(4m)(5n) + (5n)^2\} + \{(5m)^2 + 2(5m)(4n) + (4n)^2\}$
$= (16m^2 + 40mn + 25n^2) + (25m^2 + 40mn + 16n^2)$
$= (16m^2 + 25m^2) + (40mn + 40mn) + (25n^2 + 16n^2)$
$= 41m^2 + 80mn + 41n^2$

(v) $(2.5p - 1.5q)^2 - (1.5p - 2.5q)^2$
$= \{(2.5p)^2 - 2(2.5p)(1.5q) + (1.5q)^2\} - \{(1.5p)^2 - 2(1.5p)(2.5q) + (2.5q)^2\}$
$= (6.25p^2 - 7.5pq + 2.25q^2) - (2.25p^2 - 7.5pq + 6.25q^2)$
$= 6.25p^2 - 7.5pq + 2.25q^2 - 2.25p^2 + 7.5pq - 6.25q^2$
$= (6.25p^2 - 2.25p^2) + (7.5pq - 7.5pq) + (2.25q^2 - 6.25q^2)$
$= 4p^2 - 4q^2$

(vi) $(ab + bc)^2 - 2ab^2c = \{(ab)^2 + 2(ab)(bc) + (bc)^2\} - 2ab^2c$
$= (a^2b^2 + 2ab^2c + b^2c^2) - 2ab^2c$
$= a^2b^2 + (2ab^2c - 2ab^2c) + b^2c^2$ (Combining the like terms)
$= a^2b^2 + b^2c^2$

(vii) $(m^2 - n^2m)^2 + 2m^3n^2 = \{(m^2)^2 - 2(m^2)(n^2m) + (n^2m)^2\} + 2m^3n^2$
$= (m^4 - 2n^2m^3 + n^4m^2) + 2m^3n^2$
$= m^4 + (2m^3n^2 - 2n^2m^3) + n^4m^2$ (Combining the like terms)
$= m^4 + n^4m^2$

Q.5 Show that :

(i) $(3x + 7)^2 - 84x = (3x - 7)^2$ **(ii) $(9p - 5q)^2 + 180pq = (9p + 5q)^2$**

(iii) $\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2$ **(iv) $(4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2$**

(v) $(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$

Sol. (i) $\mathbf{(3x + 7)^2 - 84x = (3x - 7)^2}$

L.H.S. $= (3x+ 7)^2 - 84x$

$= \{(3x)^2 + 2(3x)(7) + (7)^2\} - 84x$

$= 9x^2 + (42x - 84x) + 49$ (Combining the like terms)

$= 9x^2 - 42x + 49$ .....(1)

R.H.S. $= (3x - 7)^2$

$= (3x)^2 - 2(3x)(7) + (7)^2$

$= 9x^2 - 42x + 49$ .....(2)

From equation (1) and (2),

$(3x + 7)^2 - 84x = (3x - 7)^2$

(ii) $\mathbf{(9p - 5q)^2 + 180pq = (9p + 5q)^2}$

L.H.S. $= (9p - 5p)^2 + 180\,pq$

$= \{(9p)^2 - 2(9p)(5q) + (5q)^2\} + 180\,pq$

$= (81p^2 - 90pq + 25q^2) + 180\,pq$

$= 81p^2 + (180pq - 90pq) + 25q^2$ (Combining the like terms)

$= 81p^2 + 90pq + 25q^2$ .....(1)

R.H.S. $= (9p + 5q)^2$

$= (9p)^2 + 2(9p)(5q) + (5q)^2$

$= 81p^2 + 90pq + 25q^2$ .....(2)

From equation (1) and (2)

$(9p - 5q)^2 + 180pq = (9p + 5q)^2$

(iii) $\mathbf{\left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn = \frac{16}{9}m^2 + \frac{9}{16}n^2}$

L.H.S. $= \left(\frac{4}{3}m - \frac{3}{4}n\right)^2 + 2mn$

$= \left(\frac{4}{3}m\right)^2 - 2\left(\frac{4}{3}m\right)\left(\frac{4}{3}n\right) + \left(\frac{3}{4}n\right)^2 + 2mn$

$= \frac{16}{9}m^2 - 2mn + \frac{9}{16}n^2 + 2mn$

$= \frac{16}{9}m^2 - (2mn - 2mn) + \frac{9}{16}n^2$ (Combining the like terms)

$= \frac{16}{9}m^2 + \frac{9}{16}n^2 = \text{R.H.S.}$

(iv) $\mathbf{(4pq + 3q)^2 - (4pq - 3q)^2 = 48pq^2}$

L.H.S. $= (4pq + 3q)^2 - (4pq - 3q)^2$

$= \{(4pq)^2 + 2(4pq)(3q) + (3q)^2\} - \{(4pq)^2 - 2(4pq)(3q) + (3q)^2\}$

$= (16p^2q^2 + 24pq^2 + 9q^2) - (16p^2q^2 - 24pq^2 + 9q^2)$

$= 16p^2q^2 + 24pq^2 + 9q^2 - 16p^2q^2 + 24pq^2 - 9q^2$

$= (16p^2q^2 - 16p^2q^2) + (24pq^2 + 24pq^2) + (9q^2 - 9q^2)$ (Combining the like terms)

$= 48pq^2 = \text{R.H.S.}$

(v) **$(a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a) = 0$**

L.H.S. $= (a - b)(a + b) + (b - c)(b + c) + (c - a)(c + a)$

$= a^2 - b^2 + b^2 - c^2 + c^2 - a^2$ (Using identity III)

$= (a^2 - a^2) + (b^2 - b^2) + (c^2 - c^2)$ (Combining the like terms)

$= 0 =$ R.H.S.

Q.6 Using identities, evaluate:

(i) 71^2 **(ii) 99^2** **(iii) 102^2** **(iv) 998^2** **(v) 5.2^2**

(vi) 297×303 **(vii) 78×82** **(viii) 8.9^2** **(ix) 1.05×9.5**

Sol. **(i)** **71^2** $= (70 + 1)^2$

$= (70)^2 + 2(70)(1) + (1)^2$ (Using Identity I)

$= 4900 + 140 + 1 = 5041$

(ii) **99^2** $= (100 - 1)^2$

$= (100)^2 - 2(100)(1) + (1)^2$ (Using Identity II)

$= 10000 - 200 + 1 = 9801$

(iii) **102^2** $= (100 + 2)^2$

$= (100)^2 + 2(100)(2) + (2)^2$ (Using Identity I)

$= 10000 + 400 + 4 = 10404$

(iv) **998^2** $= (1000 - 2)^2$

$= (1000)^2 - 2(1000)(2) + (2)^2$ (Using Identity II)

$= 1000000 - 4000 + 4 = 996004$

(v) **5.2^2** $= (5 + 0.2)^2$

$= (5)^2 + 2(5)(0.2) + (0.2)^2$ (Using Identity I)

$= 25 + 2 + 0.04 = 27.04$

(vi) **297×303** $= (300 - 3) \times (300 + 3)$

$= (300)^2 - (3)^2$ (Using Identity III)

$= 90000 - 9 = 89991$

(vii) **78×82** $= (80 - 2)(80 + 2)$

$= (80)^2 - (2)^2$ (Using Identity III)

$= 6400 - 4 = 6396$

(viii) **8.9^2** $= (9 - 0.1)^2$

$= (9)^2 - 2(9)(0.1) + (0.1)^2$ (Using Identity II)

$= 81 - 1.8 + 0.01 = 79.21$

(ix) **1.05 × 9.5** $= \frac{1}{10} \times 10.5 \times 9.5$

$= \frac{1}{10}(10+0.5)\times(10-0.5)$

$= \frac{1}{10}\times\{(10^2-(0.5)^2\}$ (Using Identity III)

$= \frac{1}{10}\times(100-0.25) = \frac{1}{10}\times 99.75 = 9.975$

Q.7 Using $a^2 - b^2 = (a + b)(a - b)$, find
(i) $51^2 - 49^2$ (ii) $(1.02)^2 - (0.98)^2$ (iii) $153^2 - 147^2$ (iv) $12.1^2 - 7.9^2$

Sol. **(i)** $51^2 - 49^2 = (51 + 49)(51 - 49)$
$= (100)(2) = 200$

(ii) $(1.02)^2 - (0.98)^2 = (1.02 + 0.98)(1.02 - 0.98)$
$= (2)(0.04) = 0.08$

(iii) $153^2 - 147^2 = (153 + 147)(153 - 147)$
$= (300)(6) = 1800$

(iv) $12.1^2 - 7.9^2 = (12.1 + 7.9)(12.1 - 7.9)$
$= (20)(4.2) = 84$

Q.8 Using $(x + a)(x + b) = x^2 + (a + b)x + ab$, find
(i) 103 × 104 (ii) 5.1 × 5.2 (iii) 103 × 98 (iv) 9.7 × 9.8

Sol. **(i)** **103 × 104** $= (100 + 3) \times (100 + 4)$
$= (100)^2 + (3 + 4)(100) + (3)(4)$
$= 10000 + 700 + 12 = 10712$

(ii) **5.1 × 5.2** $= (5 + 0.1) \times (5 + 0.2)$
$= (5)^2 + (0.1 + 0.2)(5) + (0.1)(0.2)$
$= 25 + 1.5 + 0.02 = 26.52$

(iii) **103 × 98** $= (100 + 3) \times (100 - 2)$
$= (100 + 3) \times \{100 + (-2)\}$
$= (100)^2 + \{3 + (-2)\}(100) + (3)(-2)$
$= 10000 + 100 - 6 = 10094$

(iv) **9.7 × 9.8** $= (10 - 0.3)(10 - 0.2)$
$= (10)^2 - (0.3 + 0.2)(10) + (0.3)(0.2)$
$= 100 - 5 + 0.06 = 95.06$

TRY THESE

Q.1 Find the value of the expression 2y – 5 for the other given values of y.

Sol. When $y = 5$,

$2y - 5 = 2(5) - 5 = 10 - 5 = 5$

When $y = -3$

$2y - 5 = 2(-3) - 5 = -6 - 5 = -11$

When $y = \frac{5}{2}$

$$2y - 5 = 2\left(\frac{5}{2}\right) - 5 = 5 - 5 = 0$$

When $y = \frac{-7}{3}$,

$$2y - 5 = 2\left(\frac{-7}{3}\right) - 5 = \frac{-14}{3} - 5 = \frac{-29}{3}, \text{ etc.}$$

Q.2 Give five examples of expressions containing one variable and five examples of expressions containing two variables.

Sol. Five example of expressions containing one variable

$x + 1$, $2x + 4$, $3y^2$, $\frac{3}{2}z - 7$, $t^3 + 9$

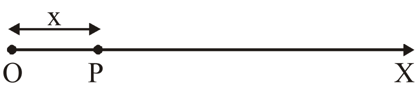
Five examples of expressions containing two variables

$xy + 3$, $x^2y - 5$, $x^3y^3 + 9$, $x^2y^2 + xy + 1$, $x^2 + y^2 + 2xy$

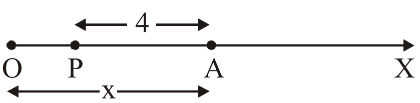
Q.3 Show on the number line.

x, x – 4, 2x + 1, 3x – 2

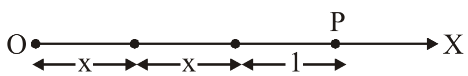
Sol. (i) x



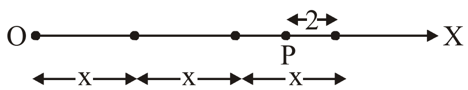
(ii) $x - 4$



(iii) $2x + 1$



(iv) $3x - 2$



Q.4 Identify the coefficient of each term in the expression $x^2y^2 - 10x^2y + 5xy^2 - 20$.

Sol.

Term Number	Term	Coefficient of term
First	x^2y^2	1
Second	$-10x^2y$	–10
Third	$5xy^2$	5
Fourth	–20	–20

Q.5 Classify the following polynomials as monomials, binomials, trinomials :
$-z + 5$, $x + y + z$, $y + z + 100$, $ab - ac$, 17

Sol.

Monomial	Binomial	Trinomial
17	$-z + 5$	$x + y + z$
	$ab - ac$	$y + z + 100$

Q.6 Construct

(a) 3 binomials with only x as a variable
(b) 3 binomials with x and y as variables;
(c) 3 monomial with x and y as variables;
(d) 2 polynomials with 4 or more terms.

Sol. **(a)** $x^2 - 4$
$x^3 - 8$
$x^4 - 1$

(b) $x + y$
$x - y$
$x^2 + y^2$

(c) $x\ y$
$x^2 y^2$
x^3y^3

(d) $x + y + z + 10$
$x + y + z + t - 6$

Q.7 Write two terms which are like

(i) 7xy **(ii) $4mn^2$** **(iii) $2l$**

Sol.
(i) 7xy : 2 like terms are $-4xy$ and xy
(ii) $4mn^2$: 2 like terms are mn^2 and $-9mn^2$.
(iii) $2l$: 2 like terms are l and $5l$.

Q.8 Can you think of two more such situations, where we may need to multiply algebraic expressions? [Hint : Think of speed and time;
Think of interest to be paid, the principal and the rate of simple interest; etc.]

Sol. (i) Distance = Speed × Time

(ii) $$\text{Simple Interest} = \frac{\text{Principle} \times \text{Rate of simple Interest per annum} \times \text{Time in years}}{100}$$

Q.9 Find $4x \times 5y \times 7z$.
First find $4x \times 5y$ and multiply it by $7z$; or first find $5y \times 7z$ and multiply it by $4x$.
Is the result the same ? What do you observe ?
Does the order in which you carry out the multiplication matter ?

Sol. $4x \times 5y \times 7z$

$$4x \times 5y = (4 \times 5) \times (x \times y) = 20 \times (xy) = 20xy$$

$$(4x \times 5y) \times 7z = 20xy \times 7z = (20 \times 7) \times (xy \times z) = 140 \times (xyz) = 140xyz \quad ..... (1)$$

$$5y \times 7z = (5 \times 7) \times (y \times z) = 35 \times (yz) = 35yz$$

$$4x \times (5y \times 7z) = 4x \times 35yz = (4 \times 35) \times (x \times yz) = 140 \times (xyz) = 140\,xyz \quad ..... (2)$$

We observe from eqns. (1) and (2) that the result is the same. It shows that the order in which we carry out the multiplity does not matter.

Q.10 Find the product :
(i) $2x(3x + 5xy)$ (ii) $a^2(2ab - 5c)$

Sol. (i) $2x(3x + 5xy)$ $= (2x) \times (3x) + (2x) \times (5xy) = 6x^2 + 10x^2y$

(ii) $a^2(2ab - 5c)$ $= (a^2) \times (2ab) - (a^2) \times (5c) = 2a^3b - 5a^2c$

Q.11 Find the product : $(4p^2 + 5p + 7) \times 3p$.

Sol. $(4p^2 + 5p + 7) \times 3p$ $= (4p^2 \times 3p) + (5p \times 3p) + (7 \times 3p) = 12p^3 + 15p^2 + 21p$

Q.12 Show that $a^2 + 3a + 2 = 132$ is not true for $a = -5$ and for $a = 0$.

Sol. For $a = -5$

L.H.S. $= a^2 + 3a + 2 = (-5)^2 + 3(-5) + 2 = 25 - 15 + 2 = 12$

R.H.S. $= 132$

$\because$ L.H.S. $\neq$ R.H.S.

$\therefore$ It is not true for $a = -5$

For $a = 0$

L.H.S. $= a^2 + 3a + 2 = (0)^2 + 3(0) + 2 = 0 + 0 + 2 = 2$

R.H.S. $= 132$

$\because$ L.H.S. $\neq$ R.H.S.

$\therefore$ It is not true for $a = 0$

Q.13 Put – b in place of b in identity (I). Do you get identity (II)?

Sol. Identity (I) is $(a + b)^2 = a^2 + 2ab + b^2$ ..... (I)

Put – b in place of b in (I), we get

$(a + (-b))^2 = a^2 + 2a(-b) + (-b)^2$

$\Rightarrow$ $(a - b)^2 = a^2 - 2ab + b^2$ which is identity (II). So yes! we get identity (II).

Q.14 1. Verify identity (IV), for a = 2, b = 3, x = 5.

2. Consider, the special case of identity (IV) with a = b, what do you get? Is it related to identity (I)?

3. Consider, the special case of identity (IV) with a = – c and b = – c. What do you get? Is it related to identity (II)?

4. Consider the special case of identity (IV) with b = –a. What do you get? Is it related to identity (III)?

Sol. 1. Identity (IV) is $(x + a)(x + b) = x^2 + (a + b)x + ab$

For a = 2, b = 3, x = 5

L.H.S. $= (x + a)(x + b)$

$= (5 + 2)(5 + 3)$

$= (7)(8) = 56$

R.H.S. $= x^2 + (a + b)x + ab$

$= (5)^2 + (2 + 3)(5) + (2)(3)$

$= 25 + 25 + 6 = 56$

L.H.S. = R.H.S.

The identity is verified.

2. Identity (IV) is $(x + a)(x + b) = x^2 + (a + b)x + ab$ .....(1)

Put a = b in (1),

Then (1) becomes

$\Rightarrow$ $(x + a)(x + a) = x^2 + (a + a)x + (a)(a)$

$(x + a)^2 = x^2 + 2ax + a^2$

It is actually identity I with a= x and b = a.

3. Identity (IV) is $(x + a)(x + b) = x^2 + (a + b)x + ab$ .....(1)

Put a = –c and b = –c

Then (1) becomes

$(x - c)(x - c) = x^2 + (-c - c)x + (-c)(-c)$

$= (x - c)^2$

$= x^2 - 2cx + c^2$

It is actually identity II with a= x and b = c.

4. Identity (IV) is $(x + a)(x + b) = x^2 + (a + b)x + ab$ .....(1)

Put b = –a

Then (1) becomes

$(x + a)(x + a) = x^2 + (a - a)x + a(-a)$

$= (x + a)(x - a)$

$= x^2 - a^2$

It is actually identity III with a= x and b = a.

CONCEPT APPLICATION LEVEL - II

SECTION - A

➢ **FILL IN THE BLANKS :**

Q.1 Find the product of $\frac{1}{4}ab$ and $-8a^2b^2$ ....

Q.2 The length and breadth of a rectangular paper are x cm and (10 – x) cm respectively. Find the area of the paper ....

Q.3 Find the value of $x^2 + 6x + 9$ for $x = 4$ ....

Q.4 Find the value of $(2x + 5)(2x - 5)$ ....

Q.5 Find the value of $(x + 5)^2 - 20x$ for $x = 6$ ....

Q.6 Write the coefficient of y^2 in $3xy^3$

Q.7 Find the product $(1 + x)(1 - x)(1 + x^2)$ ....

Q.8 Are $3x^2yz$ and $-3zyx^2$ like terms ? ....

Q.9 Find the product of the coefficients of x in $-5x^2yz$ and $2xy$ ....

Q.10 $(-x + a)(-x + b)$ is equal to ....

Q.11 .... are a combination of terms connected by the operations of addition, subtraction, multiplication or division.

Q.12 A .... may be negative or positive depending upon the sign of the ....

Q.13 The numerical factor of the term is called ....

Q.14 In expression $2x^2 + 4x$, the coefficient of x^2 is .... and coefficient of x is ....

Q.15 $5x + 4y$ is an expression having .... terms.

Q.16 An algebraic expression is called a ....if there is only one term in it.

Q.17 Trinomial is an algebraic expression with .... terms.

Q.18 $(-5\ ab^2c) \times (3a^3bc^2d) =$ ....

Q.19 Each term in an algebraic expression is a product of one or more numbers, numerical. These numbers are called the .... of that term.

Q.20 $x^2 + x - 56 = (x + 8)$ (.............................)

Q.21 Every .... polynomial has one and only one zero.

Q.22 $-3x^2yz \times \frac{1}{xyz} =$ _____________

Q.23 $4a^2bc \times$ _________________ $= 0$

Q.24 $4pqr(p^2 - q^2 + r^2) = 4p^3qr -$ ____________ $+ 4pqr^3$

Q.25 $3mn(m - n) + 2mn(n - m) = m^2n -$ _______________ .

Q.26 The value of $3x^2(x^2 - 2x + 1)$ for $x = -1$ is ___________________ .

Q.27 $34^2 - 6^2 =$ ____________ $\times$ ____________

Q.28 $(20 + 8)(20 - 8) =$ ____________ – ____________

Q.29 $(3a + 7)(3a + 8) = (3a)^2 + (7 + 8)(3a) +$ ____________

Q.30 $(6x - 7y)^2 =$ ____________ $- 2 \times 6x \times 7y +$ ____________

Q.31 $(4p^2q + 6qr)^2 = 16p^4q^2 +$ ____________ $+ 36q^2r^2$

SECTION - B

➢ **TRUE / FALSE**

Q.1 Zero may be a zero of a polynomial.

Q.2 If p(x) is a polynomial of degree ≥ 1 and 'a' is any real number then (x + a) is a factor of polynomial p(x), if p (–a) = 0.

Q.3 In the term 5ab, 5, a and b are the factors of this term.

Q.4 $(3y^2 + 3xyz) - (2x^2 - 3y^2 + 4z^2 - xyz)$ is $-2x^2 + y^2 - z^2 + xyz$.

Q.5 A constant term contains only variables.

Q.6 Only like terms can be added or subtracted.

Q.7 If the polynomial is $a_0 + a_1 x + a_2x + ... + a_nx^n$, $a_n \neq 0$, then its degree is n.

Q.8 Degree of a polynomial is a rational number.

Q.9 If the polynomial is a_0 $(a_0 \neq 0)$, then it is called a zero polynomial of 0 degree.

Q.10 There are 5 terms in the algebraic expression $4x^3 - 3x^2 + 2x - 9$.

Q.11 The coefficient of x in $2x^3 + 7x^2 - x$ is 1.

Q.12 The coefficient of y^2 in $4y^4 - 3y^3 + 3y^2 - 2$ is 3.

Q.13 The value of $-2a^2 + 3a - 6$ at a = – 1 is 11.

Q.14 The degree of the polynomial $9x^7 - \frac{3}{4}x^4 + 11x$ is 7.

Q.15 The product of a monomial and another monomial is always a monomial.

Q.16 The product of two binomials is always a binomial.

Q.17 The product of $4p^2q$ and pq^2 is $4p^2q^3$.

Q.18 The product of xyz and $-\frac{1}{xyz}$ is – 1.

Q.19 Distributive law and commutative law holds good for multiplication of polynomials.

SECTION - C

➢ **MULTIPLE CHOICE QUESTIONS**

Q.1 The coefficient of x^0 in $3x^3 - 4x^2 + 7x - 2$ is :
(A) 7 (B) 2 (C) –2 (D) 0

Q.2 The number of like terms in $4x^2y - 6xy^2 + 3x^2y - 2yx + 7xyz$ is :
(A) 2 (B) 3 (C) 4 (D) None

Q.3 The vlaue of expression $4a^2b - 2ab^2 + 7ab - 3$ at a = –1, b = 2 is
(A) – 27 (B) 1 (C) 27 (D) – 1

Q.4 The sum of $-2a - b + 3c - d$ and $2a + 4b + 6c$ is
(A) Monomial (B) Binomial (C) Trinomial (D) Polynomial with 4 terms

Q.5 The product of $-7x^2yz$, $-13y^2$ and yz is :
(A) $91\ xy^4z^2$ (B) $91\ xy^2$ (C) $91x^2y^4z^2$ (D) $-91x^2yz^2$

Q.6 The product of $-7xy$ and $x^2 - 4y^2$ is
(A) $7x^2y^2 - 28\ xy^3$ (B) $-7x^3y + 28xy^3$ (C) $-7x^2y + 28\ x^2y^2$ (D) $-7x^3 - 28xy^3$

Q.7 The value of the expression $4xy\ (x^2 - y^2)$ at $x = -1$ and $y = -1$ is
(A) 4 (B) –1 (C) –4 (D) 0

Q.8 On simplifying $3ab\ (a^2 - b^2) - 3b\ (a^3 - ab^2 + 4ab)$, we get
(A) $-12ab^2$ (B) 0 (C) $12a^2b$ (D) $3a^2b^2$

Q.9 The value of the expression $3xy\ (x^2 - xy + y^2)$ at $x = 1$, $y = -1$ is
(A) –3 (B) – 9 (C) 3 (D) 1

Q.10 On simplifying $4pq\ (p^2 + q^2) - 4pq\ (p^2 - q^2)$, we get
(A) 0 (B) $8p^3q$ (C) $8pq^3$ (D) $-8pq^3$

Q.11 The number of terms in the product of $(x - 4)$ and $(x^2 - 7x + 12)$ is
(A) 4 (B) 6 (C) 5 (D) 2

Q.12 The area of a rectangle is length × breadth. If the length is $(x + 3)$ units and breadth is $(y - 3)$ units the area is :
(A) $xy - 3y + 3x - 9$ (B) $xy + 3y - 3x - 9$ (C) $xy + 3y + 3x - 9$ (D) $xy - 3y - 3x - 9$

Q.13 The volume of a box is given by the formula length × breadth × height. If the length is 4x units, breath is $3y^2$ units and height is $2x^2y$ units the volume is :
(A) $12x^2y^2$ (B) $24xy^3$ (C) $24x^3y^2$ (D) $24x^3y^3$

Q.14 The middle term in the expression $(2y - 3z)^2$ is
(A) 12yz (B) –6yz (C) –12yz (D) 6yz

Q.15 The vlaue of $13.1^2 - 6.9^2$ is
(A) 124 (B) 12.4 (C) 1.24 (D) 1124

Q.16 The degree of $4x^2y + x^2 - 2y^2$ is
(A) 2 (B) 3 (C) 4 (D) – 2

Q.17 What must be added to $2x^2 - 3x - 8$ to get $3x^2 + x + 6$?
(A) $x^2 + 4x + 14$ (B) $-x^2 - 4x - 14$ (C) $5x^2 - 2x - 2$ (D) $x^2 - x - 2$

Q.18 What must be subtracted from $x^3 + 3x - 9$ to get $3x^3 + x^2 + 3$?
(A) $4x^3 + 4x^2 - 6$ (B) $4x^3 + x^2 + 3x - 6$
(C) $-2x^3 - x^2 + 3x - 12$ (D) $2x^3 - x^2 - 3x + 12$

Q.19 If a = b = 2, then the value of $(a - b)(a^2 + ab - b^2)$ will be
(A) 2 (B) 0 (C) 1 (D) 48

Q.20 $\frac{2}{3}xy \times \frac{3}{4}xz$ is
(A) $\frac{1}{12}x^2y$ (B) $\frac{1}{2}xyz$ (C) $\frac{1}{2}x^2yz$ (D) $\frac{1}{4}x^2y$

Q.21 $(-3ab) \times (-2a^2b)$ is
(A) $-6ab^2$ (B) $6a^3b^2$ (C) $6a^3b^3$ (D) $-6a^2b^2$

Q.22 $25x^8y^9z^5 \times (-2xyz)^2$ is
(A) $50x^9y^{10}z^6$ (B) $100\,x^{10}y^{11}z^7$ (C) $-100x^{10}y^{11}z^7$ (D) $-50x^9y^{10}z^6$

Q.23 The true statement is
(A) $3x = 3 + x$ (B) $2(x + 5) = 2x + 5$
(C) $4(x^2 - 3) = 4x^2 - 12$ (D) $x \times (-5) = x - 5$

Q.24 The product of $\left(\frac{3}{2}xyz - \frac{9}{4}xy^2z^3\right)$ and $\left(\frac{-8}{27}xyz\right)$ is :
(A) $\frac{4}{9}x^2y^2z^2 + \frac{2}{3}x^3y^3z^3$ (B) $-\frac{4}{9}x^2y^2z^2 + \frac{2}{3}x^2y^3z^4$
(C) $\frac{4}{9}xy^2z^3 + \frac{2}{3}x^2y^3z^2$ (D) $-\frac{4}{9}x^2y^3z^4 - \frac{8}{3}x^2y^2z^2$

Q.25 $(3x - 4)(2x + 7)$ is
(A) $6x^2 + 3x - 28$ (B) $6x^2 + 13x - 28$ (C) $6x^2 + 29x - 28$ (D) $6x^2 - 29x - 28$

Q.26 $\left(\frac{1}{2}x^2 + y^2\right)\left(x^2 - \frac{1}{2}y^2\right)$ is
(A) $\frac{1}{2}x^4 - \frac{1}{2}y^4$ (B) $\frac{1}{2}x^4 - \frac{3}{4}x^2y^2 + \frac{1}{2}y^4$
(C) $\frac{1}{2}x^4 + \frac{3}{4}x^2y^2 - \frac{1}{2}y^4$ (D) $\frac{1}{2}x^2 + \frac{3}{4}x^2y^2 - \frac{1}{2}y^2$

Q.27 When a = 3, b = 2, then $(a - b)(2a^2 - 3ab + b^2)$ is
(A) 4 (B) –4 (C) 20 (D) –20

Q.28 $(y + 5)(y - 3)$ is
(A) $y^2 - 8y - 15$ (B) $y^2 + 8y - 15$ (C) $y^2 + 2y - 15$ (D) $y^2 - 2y - 15$

Q.29 Square of $2x^2 - 3y^2$ is
(A) $4x^2 - 9y^4$ (B) $4x^4 + 9y^4 - 12x^2y^2$ (C) $4x^4 + 9y^4 + 12x^2y^2$ (D) $4x^4 + 9y^4$

Q.30 $\left(\frac{2}{3}x^2 - \frac{1}{2}y^2\right)\left(\frac{2}{3}x^2 + \frac{1}{2}y^2\right)$ is
(A) $\frac{4}{9}x^4 + \frac{1}{4}y^4$ (B) $\frac{4}{9}x^2 - \frac{1}{4}y^2$ (C) $\frac{4}{9}x^4 - \frac{1}{4}y^4$ (D) $\frac{4}{9}x^4 - \frac{1}{4}y^4 - \frac{2}{3}x^2y^2$

Q.31 The value of $\frac{7.87 \times 7.87 - 1.72 \times 1.72}{6.15}$ is
(A) 9.59 (B) 10 (C) 6.15 (D) 6.45

Q.32 $6y^4 \div (-2y^3)$ is
(A) $3y$ (B) $-3y$ (C) $3y^3$ (D) $-3y^3$

Q.33 $(-72x^2y^3) \div (-8xy)$ is
(A) $-9xy$ (B) $-9xy^2$ (C) $9xy^2$ (D) $9xy$

Q.34 $(8x^2y^2 + 6xy^2 - 10x^2y^3) \div (2xy)$ is
(A) $4xy + 3y - 5xy^2$ (B) $4xy^2 - 3y - 5xy^2$ (C) $4xy - 3y + 5y^2$ (D) $4xy^2 + 3x - 5x^2y$

Q.35 The remainder obtained when $t^4 - 3t^3 + t + 5$ is divided by $t - 1$ is :
(A) -4 (B) 4 (C) 1 (D) 5

Q.36 The product of two expressions is $x^5 + x^3 + x$. If one of them is $x^2 + x + 1$, find the other.
(A) $x^3 - x^2 + x$ (B) $x^3 + x^2 + x$ (C) $x^3 + x^2$ (D) $x^3 - x^2$

Q.37 Find the value of k, so that $x - 3$ is a factor of $3x^2 - 11x + k$.
(A) 6 (B) 3 (C) 9 (D) 27

Q.38 In the expression $-7x^2y + 3xy + 3$, the coefficient of x^2 is
(A) 7 (B) 7y (C) $-7y$ (D) $-7xy$

Q.39 The sum of $a^2 + b^2$ and $a + b$ is :
(A) $a^3 + b^3$ (B) $2a^2 + 2b^2$ (C) $a^2 + b^2 + a + b$ (D) none of these

Q.40 Which of the following is a pair of unlike terms ?
(A) $4ab, -3ba$ (B) $6a^3b^3c, -3cb^3a^3$ (C) $7ab, -ab$ (D) $3a^2b, -5ab^2$

Q.41 When the expressions $5x^2 - 8xy$ and $-3x^2 + 2xy$ are added, we get :
(A) $2x^2 - 6xy$ (B) $8x^2 - 10xy$ (C) $-2x^2 + 6xy$ (D) $2x^2 + 6xy$

Q.42 When $12x + 10y$ is subtracted from $-13x + 7y$, we get :
(A) $25x + 3y$ (B) $-25x - 3y$ (C) $-25x + 3y$ (D) $25x - 3y$

Q.43 Subtract $1 + x^2 + y^2$ from the sum of $x^2 - y^2$ and $1 - x^2 - y^2$.
(A) $-x^2 - 3y^2$ (B) $x^2 + y^2$ (C) $3x^2 - 3y^2$ (D) 2

Q.44 Subtract the sum of $3x - 4y + z$ and $1 - 2x + y$ from the sum of $3y + z + 6$ and $x - 2y + 3$.
(A) $4y + 8$ (B) 0 (C) $2x + 2y + 2z + 10$ (D) $2x + 2y + 2z$

Q.45 The value of $\left(\frac{1}{2}xyz\right)(-4xy^2)$ is :
(A) $2x^2y^2z^2$ (B) $-2x^2y^3z$ (C) $2x^2y^3z^2$ (D) $-2xyz$

Q.46 The value of $x^{52} \times x^{-13} \times 0$ is :
(A) 0 (B) x^{39} (C) x^{65} (D) $-x^{39}$

Q.47 The value of $25x^3y^2z$ for $x = 1, y = 2$ and $z = 3$ is :
(A) 600 (B) 500 (C) 300 (D) none of these

Q.48 When the product of $x^5 \times x^2 \times x^6$ is expressed as a monomial, we get :
(A) x^{60} (B) x^{13} (C) $3x^{13}$ (D) $3x^{60}$

Q.49 The product of $-3x^2y$, $4xy^2$ and $2xyz$ is :
(A) $-9x^4y^4z$ (B) $24x^4y^4z$ (C) $-24x^4y^4z$ (D) $3x^4y^4z$

Q.50 In its simplest form $2x(1 - 3y) - x(y - 3)$ is :
(A) $7x - 5xy$ (B) $5x - 7xy$ (C) $5x + 7xy$ (D) none of these

Q.51 Which of the following is the same as $(x + 3)(2x - 5)$?
(A) $2x^2 + x + 15$ (B) $-2x^2 + x + 15$ (C) $2x^2 + x - 15$ (D) $2x^2 - x + 15$

Q.52 The product $(y^4 - x^4)(y^2 + x^2)$ is :
(A) $y^6 + x^2y^4 - x^4y^2 - x^6$ (B) $x^6 - x^2y^4 + x^4y^2 - y^6$
(C) $y^6 - x^2y^4 + x^4y^2 + x^6$ (D) $y^6 + x^2y^4 - x^4y^2 + x^6$

Q.53 When $(7x^2 + 9x - x^o)$ is multiplied by $15x^2$, we get
(A) $105x^4 + 135x^3$ (B) $105x^4 + 135x^3 - 15x^2$
(C) $105x^4 + 135x^3 - 1$ (D) $105x^4 + 135x^3 + 15x^2$

Q.54 $(2x + 5y)^2$ is equal to :
(A) $4x^2 + 25y^2 + 10xy$ (B) $4x^2 - 25y^2 + 20xy$ (C) $4x^2 + 25y^2 + 20xy$ (D) none of these

Q.55 $(1 - 2x)^2$ is equal to :
(A) $1 + 4x^2 + 4x$ (B) $1 - 4x^2 - 4x$ (C) $-1 + 4x^2 - 4x$ (D) $1 + 4x^2 - 4x$

Q.56 On simplification $(2x + 3y)^2 - (2x - 3y)^2$, we get :
(A) $4x^2 - 7y^2$ (B) $4x^2 - 6y^2$ (C) $24xy$ (D) $4xy$

Q.57 If $x - \frac{1}{x} = 8$, then the value of $x^2 + \frac{1}{x^2}$ is :
(A) 66 (B) 64 (C) 62 (D) 68

Q.58 Using the identities, evaluuate :
$$\frac{5.27 \times 5.27 - 0.27 \times 0.27}{5.54}$$
(A) 5 (B) $\frac{5}{5.54}$ (C) 25 (D) $\frac{25}{5.54}$

Q.59 Find the value of x, if :
$12x = 50 \times 50 - 38 \times 38$
(A) 88 (B) 1 (C) 1900 (D) 44

Q.60 If $\left(x + \frac{1}{x}\right) = 16$, find the value of $x^2 + \frac{1}{x^2}$.
(A) 254 (B) 258 (C) 256 (D) $256 - 2x$

Q.61 If $\left(x - \frac{1}{x}\right)^2 = 81$, find the value of $x^2 + \frac{1}{x^2}$.
(A) 9 (B) 81 (C) 83 (D) 72

Q.62 If $x + \frac{1}{x} = 4$, find the value of $x^4 + \frac{1}{x^4}$.
(A) 194 (B) 196 (C) 190 (D) 184

Q.63 The product of $\left(\frac{9}{2}xyz - \frac{3}{4}xy^2z^3\right)$ and $\left(\frac{-8}{27}xyz\right)$ is :

(A) $\frac{4}{9}x^2y^2z^2 + \frac{2}{3}x^3y^3z^3$ (B) $-\frac{4}{3}x^2y^2z^2 + \frac{2}{9}x^2y^3z^4$

(C) $\frac{4}{9}xy^2z^3 + \frac{2}{3}x^2y^3z^2$ (D) $-\frac{4}{9}x^2y^3z^4 - \frac{8}{3}x^2y^2z^2$

Q.64 $(y - 5)(y + 3)$ is :
(A) $y^2 - 8y - 15$ (B) $y^2 + 8y - 15$ (C) $y^2 - 2y - 15$ (D) $y^2 - 2y + 15$

Q.65 Square of $(2x^2 + 3y^2)$ is :
(A) $4x^2 - 9y^2$ (B) $4x^4 + 9y^4 - 12x^2y^2$ (C) $4x^4 + 9y^4 + 12x^2y^2$ (D) $4x^4 + 9y^4$

Q.66 $\left(\frac{2}{3}x - \frac{1}{2}y\right)\left(\frac{2}{3}x + \frac{1}{2}y\right)$ is :

(A) $\frac{4}{9}x^4 + \frac{1}{4}y^4$ (B) $\frac{4}{9}x^2 - \frac{1}{4}y^2$ (C) $\frac{4}{9}x^4 - \frac{1}{4}y^4$ (D) $\frac{4}{9}x^4 - \frac{1}{4}y^4 - \frac{2}{3}x^2y^2$

Q.67 $(3x + 4)(2x - 7)$ is :
(A) $6x^2 - 13x - 28$ (B) $6x^2 + 13x - 28$ (C) $6x^2 + 29x - 28$ (D) $6x^2 - 29x - 28$

Q.68 $\left(\frac{1}{2}x^2 - y^2\right)\left(x^2 + \frac{1}{2}y^2\right)$ is :

(A) $\frac{1}{2}x^4 - \frac{1}{2}y^4$ (B) $\frac{1}{2}x^4 - \frac{3}{4}x^2y^2 + \frac{1}{2}y^4$

(C) $\frac{1}{2}x^4 + \frac{3}{4}x^2y^2 - \frac{1}{2}y^4$ (D) $\frac{1}{2}x^2 + \frac{3}{4}x^2y^2 - \frac{1}{2}y^2$

Q.69 When a = 3, b = 2, then $(a + b)(2a^2 - 3ab - b^2)$ is :
(A) 4 (B) – 4 (C) 20 (D) – 20

Q.70 If the polynomials $(px^3 + 4x^2 + 8x - 4)$ and $(x^3 - 4x + p)$ are divided by $(x - 3)$ then the remainder in each case is the same. Then the value of p is :
(A) –1 (B) –2 (C) 1 (D) 2

Q.71 If $x^2 + \frac{1}{x^2} = 27$, then value of $x + \frac{1}{x}$ is

(A) 9 (B) 29 (C) $\sqrt{29}$ (D) 3

Q.72 The quotient of division of $x^3 - 3x^2 + 5x - 3$ by $x^2 - 2$ is
(A) (x + 3) (B) (x – 3) (C) (x + 2) (D) (x – 2)

Q.73 Value of $\frac{991 \times 991 \times 991 + 9 \times 9 \times 9}{991 \times 991 - 991 \times 9 + 9 \times 9}$ is :

(A) 991 (B) 9 (C) 1000 (D) 991 × 9

Q.74 What should be added to $\frac{1}{x^2 - 7x + 12}$ to get $\frac{2}{x^2 - 6x + 8}$?

(A) $\frac{1}{x^2 + 5x - 16}$ (B) $\frac{1}{(x+3)(x+2)}$ (C) $\frac{4}{(x-3)(x+2)}$ (D) $\frac{1}{x^2 - 5x + 6}$

SECTION - D

➢ MORE THAN ONE CORRECT

Q.1 Which of the following expressions are polynomials ?

(A) $3x^2 - 4x + 5$ (B) $\frac{1}{2}x^2 - \frac{2}{3}x + \frac{5}{7}$ (C) $9x + 2$ (D) 2

Q.2 Which of the following expressions are not polynomials ?

(A) $\frac{2}{x} + x^3 + 2$ (B) $\frac{3x^2 - x + 1}{x^2 + 1}$ (C) $\frac{3x + 2}{x^2}$ (D) $4x^3 + 5x^{10} - 9x^8 + 1$

Q.3 Which of the following expressions are binomials and trinomials ?

(A) $\frac{3}{x^2}, 5x$ (B) $x + \frac{2}{x}, x^2 + 2x - 5$

(C) $x^2 + 2x - 5, 9x^3 + 5x$ (D) $\frac{7x}{5} - \frac{9}{8}, 3x^2 - 4x + 5$

Q.4 Which of the following terms are like terms ?

(A) $8ab, -9ab^2$ (B) $3x^2y, -4yx^2$ (C) $8xy^2, -11x^3y$ (D) $\frac{7}{4}xy, -\frac{5}{3}xy$

Q.5 Which of the following terms are unlike terms ?

(A) $\frac{3}{4}a^2bx, \frac{3}{4}ab^2x, \frac{3}{4}abx^2$ (B) $-9xy, 5$

(C) $-9x^2, -10x^2, 5x^2$ (D) None of these

Q.6 If the given expression is a complete square, then which of the following formulae we use to factorise it?

(A) $a^2 + 2ab + b^2 = (a + b)^2$ (B) $a^2 - 2ab + b^2 = (a - b)^2$

(C) $(a - b)(a + b) = (a^2 - b^2)$ (D) $(x + a)(x + b) = x^2 + (a + b)x + ab$

Q.7 Which of the following polynomials are of degree 1 and 3 ?

(A) $-4 + 5x, -5 + 7t + 6t^3$ (B) $2a + \frac{9}{4}, 9x^2 + 6x - 5$

(C) $\frac{7}{2} + 4x^2 - 3x^3, \frac{-15}{4}$ (D) $5 + 4x, \frac{7}{2} + 4x^2 - 3x^3$

Q.8 In the term $\frac{25}{3}a^2bc^3$, which of the following is/are correct ?

(A) Coefficient of $a^2 = \frac{25}{3}bc^3$ (B) Numerical coefficient $= \frac{25}{3}$

(C) Coefficient of $c^3 = \frac{25}{3}a^2b$ (D) Coefficient of $a^2bc^3 = \frac{25}{3}$

SECTION - E

➢ MATCH THE COLUMN

Q.1 Match the Column

	Column-I		Column-II
(A)	$a^3 + b^3$	(p)	$a^2 + b^2 + 2ab$
(B)	$a^3 - b^3$	(q)	$a^3 - b^3 - 3ab\,(a - b)$
(C)	$(a + b)^3$	(r)	$(a + b)\,(a^2 - ab + b^2)$
(D)	$(a - b)^3$	(s)	$a^3 + b^3 + 3ab\,(a + b)$
(E)	$(a + b)^2$	(t)	$(a - b)\,(a^2 + ab + b^2)$
(F)	$(a - b)^2$	(u)	$a^2 + b^2 - 2ab$

Q.2 Match the Column

	Column-I		Column-II
(A)	$\left(\frac{2}{3}a^2b\right)\left(\frac{-9}{4}ab^2\right)$	(p)	$\frac{-4}{9}p^5q^5$
(B)	$(-pq)\,(-2.3\,p^2q^2)\,(-0.1\,p^2q)$	(q)	$0.225\,a^4b^4c$
(C)	$(-1.5a^2b)\,(0.3ab^2)\,(-0.5\,abc)$	(r)	$\frac{-3}{2}a^3b^3c^2$
(D)	$\left(\frac{-3}{7}p^3q^2\right)\left(\frac{-14}{9}pq^2\right)\left(\frac{-2}{3}pq\right)$	(s)	$-0.23\,p^5q^4$

SECTION - F

➢ Multiple Matching Questions

Direction : Each question has statements (A, B, C, D ......) given in Column I and statements (p, q, r, s .....) in Column II. Any given statement in Column I can have correct matching with one or more statement(s) given in Column II. Match the entries in column I with entries in column II.

Q.1

	Column-I		Column-II
(A)	Monomials	(p)	$\sqrt{6ab} - \frac{a^2b}{5}$
(B)	Binomials	(q)	$a^2 + 2ab + b^2$
(C)	Trinomials	(r)	p^2q^3y
(D)	Polynomials	(s)	$3xy + 4x^2y$
		(t)	$\frac{3}{4}xy$
		(u)	$x^2 + 3xy + y^2 + 5x^2y + 4xy^2$

SECTION - G

➢ **Assertion & Reason**

Direction : Each of these questions contains as Assertion followed by Reason. Read them carefully and answer the question on the basis of following options. Your have to select the one that best describes the two statements

(A) If both **Assertion** and **Reason** are **correct** and Reason is the **correct explanation** of Assertion.

(B) If both **Assertion** and **Reason** are correct, but Reason is not **the correct explanation** of Assertion.

(C) If both **Assertion** is **correct** but **Reason** is **incorrect**.

(D) If both **Assertion** is **incorrect** but **Reason** is **correct**.

Q.1 **Assertion :** Degree of the polynomial $5x^2 + 3x + 4$ is 2.

Reason : The degree of a polynomial of one variable is the highest value of thhe exponent of the variable.

Q.2 **Assertion :** Binomials and Trinomials are multinomials.

Reason : An algebraic expression having two or more terms is called a multinomial.

Q.3 **Assertion :** In the expression $3x^2 + 7y^2 – 2xy + 4x^2 + 8xy + 9y^2$, $3x^2$, $4x^2$ are like terms, $–2xy$, $8xy$ are like terms and $7y^2$, $9y^2$ are like terms.

Reason : When the terms have same literal factors they are called unlike terms.

Q.4 **Assertion :** We should multiplyy $(–7)^{–1}$ to $\frac{-7}{4}$ to get the product as $4^{–1}$.

Reason : If $\frac{x}{y} = \left(\frac{5}{2}\right)^{-1} \times \left(\frac{8}{9}\right)^{0}$ then value of $\left(\frac{x}{y}\right)^{-2}$ is $\left(\frac{2}{5}\right)^{2}$.

ANSWER KEY

CONCEPT APPLICATION LEVEL - II

SECTION - A

Q.1	$-2a^3b^3$	Q.2	$(10x - x^2)\ cm^2$	Q.3	49	Q.4	$4x^2 - 25$
Q.5	1	Q.6	3xy	Q.7	$1 - x^4$	Q.8	yes
Q.9	$-10x^2y^2z$	Q.10	$x^2-(a+b)\ x + ab$	Q.11	Algebraic expressions		
Q.12	term, term	Q.13	coefficient	Q.14	2, 4	Q.15	two
Q.16	monomial	Q.17	three	Q.18	$-15\ a^4b^3c^3d$	Q.19	factors
Q.20	$(x - 7)$	Q.21	linear	Q.22	$-3x$	Q.23	0
Q.24	$4pq^3r$	Q.25	mn^2	Q.26	12	Q.27	40×28
Q.28	$400 - 64$	Q.29	7×8	Q.30	$36x^2, 49y^2$	Q.31	$48p^2q^2r$

SECTION - B

Q.1	True	Q.2	True	Q.3	True	Q.4	False	Q.5	False	Q.6	True	Q.7	True
Q.8	False	Q.9	False	Q.10	False	Q.11	False	Q.12	True	Q.13	False	Q.14	True
Q.15	True	Q.16	False	Q.17	False	Q.18	True	Q.19	True				

SECTION - C

Q.1	C	Q.2	A	Q.3	D	Q.4	C	Q.5	C	Q.6	B	Q.7	D
Q.8	A	Q.9	B	Q.10	C	Q.11	A	Q.12	B	Q.13	D	Q.14	C
Q.15	A	Q.16	B	Q.17	A	Q.18	C	Q.19	B	Q.20	C	Q.21	B
Q.22	B	Q.23	C	Q.24	B	Q.25	B	Q.26	C	Q.27	A	Q.28	C
Q.29	B	Q.30	C	Q.31	A	Q.32	B	Q.33	C	Q.34	A	Q.35	B
Q.36	A	Q.37	A	Q.38	C	Q.39	C	Q.40	D	Q.41	A	Q.42	B
Q.43	A	Q.44	A	Q.45	B	Q.46	A	Q.47	C	Q.48	B	Q.49	C
Q.50	B	Q.51	C	Q.52	A	Q.53	B	Q.54	C	Q.55	D	Q.56	C
Q.57	A	Q.58	A	Q.59	A	Q.60	A	Q.61	C	Q.62	A	Q.63	B
Q.64	C	Q.65	C	Q.66	B	Q.67	A	Q.68	B	Q.69	D	Q.70	A
Q.71	C	Q.72	B	Q.73	C	Q.74	D						

SECTION - D

Q.1	ABCD	Q.2	ABC	Q.3	CD	Q.4	BD	Q.5	AB
Q.6	AB	Q.7	AD	Q.8	ABCD				

SECTION - E

Q.1 (A) $\rightarrow$ r (B) $\rightarrow$ t (C) $\rightarrow$ s (D) $\rightarrow$ p (E) $\rightarrow$ p (F) $\rightarrow$ u

Q.2 (A) $\rightarrow$ r (B) $\rightarrow$ s (C) $\rightarrow$ q (D) $\rightarrow$ p

SECTION - F

Q.1 (A) $\rightarrow$ r,t (B) $\rightarrow$ p,s (C) $\rightarrow$ q (D) $\rightarrow$ u

SECTION - G

Q.1	A	Q.2	A	Q.3	C	Q.4	C