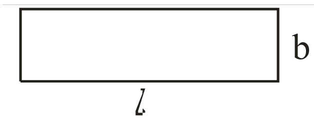
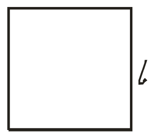
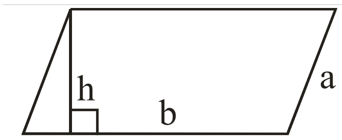
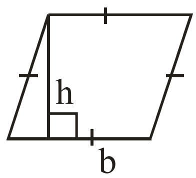
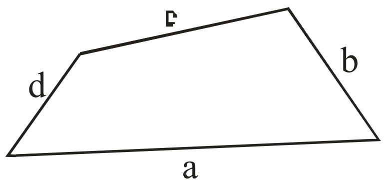
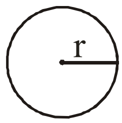
16 MENSURATION

16.1 INTRODUCTION

In earlier classes, we have already dealt that for a closed figure, the perimeter is the distance around and area is the region covered by it.

To recall, let us state some formulae, which we have already discussed in the earlier classes.

	Perimeter	**Area**	**Figure**
Rectangle	$2(l + b)$	$l \times b$	
Square	$4(l)$	l^2	
Parallelogram	2 (sum of lengths of adjacent sides $= 2(a + b)$	(base × corresponding altitude) $= b \times h$	
Rhombus	$4 \times$ side	$= b \times h$	
Quadrilateral	Sum of lengths of four sides		
Circle	$2\pi r$	πr^2	

16.2 PLANE FIGURES

We have already dealt with plane figures (Triangles, Quadrilateral and Circles) in geometry chapter. In this chapter we will deal with perimeter and area of plane figures.

16.2.1 Perimeter

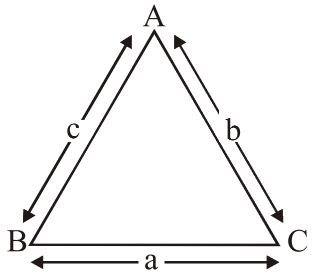
The perimeter of a plane geometrical figure is the total length of sides (or boundary) enclosing the figure. Units of measuring perimeter can be mm, cm, m, km etc.

16.2.2 Area

The area of any figure is the amount of surface enclosed within its bounding lines. Area is always expressed in square units.

16.3 TRIANGLES

A triangle is plane figure bounded by three straight lines.
Let ABC is triangle [figure] in which BC = a, AC = b and AB = c,
then perimeter of ΔABC, $2S = a + b + c$



or $S = \dfrac{a+b+c}{2}$ = semi-perimeter of ΔABC.

If we know three sides of a Δ then

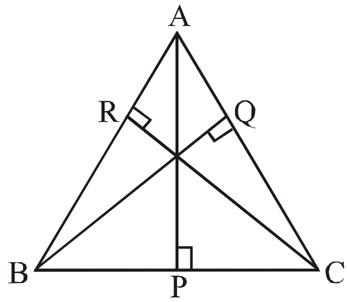
Area = $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \dfrac{a+b+c}{2}$ = Semi-perimeter.

This is known as **Heron's formula**.
If we show perpendicular AP from vertex 'A' on side BC then AP is called altitude (or height) of triangle ABC corresponding to BC.
Similarly BQ and CR are altitude of ΔABC corresponding to bases AC and AB respectively.

For any triangle ABC

Area = $\dfrac{1}{2} \times$ base $\times$ corresponding altitude

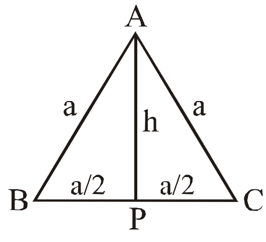
Area = $\dfrac{1}{2} BC \times AP = \dfrac{1}{2} AC \times BQ = \dfrac{1}{2} AB \times CR$.



16.3.1 Types of Triangles

(1) **Equilateral Triangle :** A triangle whose all sides are equal is called an equilateral triangle.
ABC is an equilateral triangle in

$\therefore$ AB = BC = CA = 'a' say



From ΔAPC

$$AC^2 = AP^2 + PC^2$$
$$AP^2 = AC^2 - PC^2$$
$$AP^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$$
$$AP = \frac{\sqrt{3}}{2} a = h = \frac{\sqrt{3}}{2} a$$ [height of equilateral Δ]

Area of equilateral $\Delta = \dfrac{1}{2} \times a \times \dfrac{\sqrt{3}}{2} a$

Area of equilateral $\Delta = \dfrac{\sqrt{3}}{4}a^2$ sq. units. $= \dfrac{\sqrt{3}}{4} \times \left(\dfrac{2h}{\sqrt{3}}\right)^2$ $\left[h = \dfrac{\sqrt{3}}{2}a\right]$

Area of equilateral $\Delta = \dfrac{\sqrt{3}}{4} \times \dfrac{4h^2}{\sqrt{3}\times\sqrt{3}} = \dfrac{h^2}{\sqrt{3}}$ sq. units.

Perimeter of equilateral triangle = 3 × side = 3a units.

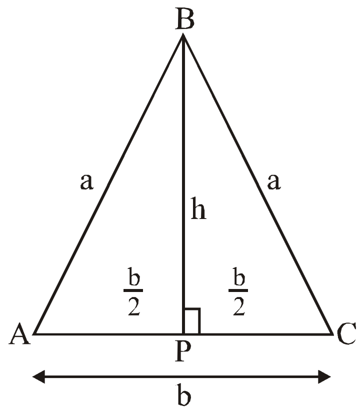
(2) **Isosceles Triangle :** A triangle whose two sides are equal is an isosceles triangle.
In ΔABC, figure let AB = BC = a say and base AC = b.
From ΔBPC

$BC^2 = BP^2 + PC^2$

$BP^2 = BC^2 - PC^2$

$BP^2 = h^2 = a^2 - \dfrac{b^2}{4}$

$\therefore \quad h = \dfrac{\sqrt{4a^2 - b^2}}{2}$ = height of ΔABC



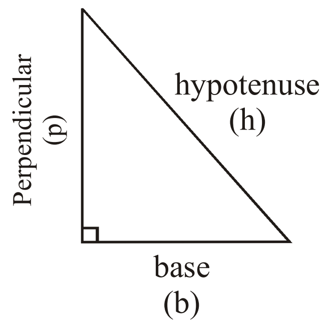
$\Rightarrow \quad$ Area of $\Delta ABC = \dfrac{1}{2} \times AC \times BP$

$= \dfrac{1}{2} \times b \times \dfrac{\sqrt{4a^2 - b^2}}{2}$

Area of $\Delta ABC = \dfrac{b}{4}\sqrt{4a^2 - b^2}$ sq. units. (where a is the equal sides of triangle)

Perimeter of ΔABC = AB + BC + CA = (2a + b) units.

(3) **Right Angled Triangle :** A triangle having one of this angles equal to 90º (right angle) is called right angled triangle. The side opposite to the right angle is called hypotenuse. In a right angled triangle.



$(\text{hypotenuse})^2 = (\text{base})^2 + (\text{perpendicular})^2$

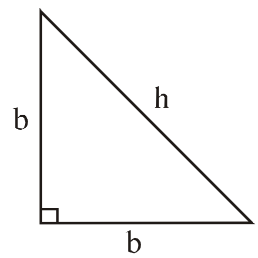
$h^2 = b^2 + p^2$

Area $= \dfrac{1}{2} \times$ base × perpendicular $= \dfrac{1}{2}bp$

(4) **Isosceles Right Angled Triangle :** An isosceles right angled triangle has two equal sides making 90º to each other.
According to figure

$(\text{hypoteneuse})^2 = b^2 + b^2$

$h^2 = 2b^2 \quad \Rightarrow \quad h = b\sqrt{2}$



base = b, height = b

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times b \times b = \frac{b^2}{2}$$

Also $b = \frac{h}{\sqrt{2}}$

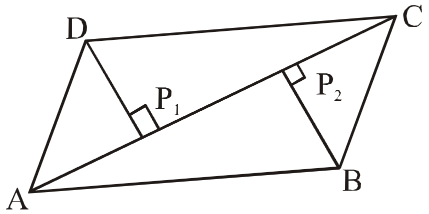
$$\therefore \quad \text{Area} = \frac{\left(\frac{h}{\sqrt{2}}\right)^2}{2} = \frac{h^2}{4}$$

Perimeter $= 2b + h = 2b + b\sqrt{2} = \sqrt{2}\,b(\sqrt{2}+1)$

16.4 QUADRILATERAL

A close figure bounded by four sides is called a quadrilateral. It has 4 included angles and the sum of 4 included angles is 360º.

Area of quadrilateral ABCD is



$$A = \frac{1}{2} \times \text{one diagonal} \times (\text{sum of } \perp^r \text{ to it from opposite vertices})$$

$$A = \frac{1}{2} \times d \times (P_1 + P_2) \qquad \text{where } d = AC.$$

If length of four sides and one of its diagonals are given, then

Area (A) of quadrilateral = Area of ΔABC + Area of ΔADC

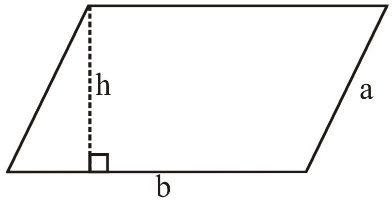
16.4.1 Types of Quadrilaterals

(1) **Parallelogram :** A quadrilateral in which opposite sides are equal and parallel is called a parallelogram.

Note : diagonals of a parallelogram bisect each other.

Area of parallelogram = Base × Corresponding height

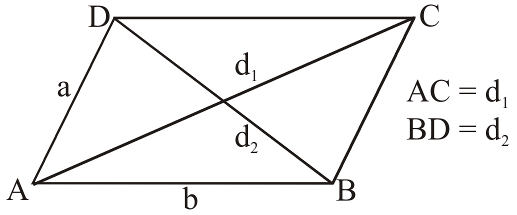
$A = b \times h$



Perimeter (P) of parallelogram = 2(a + b) where a and b are adjacent sides.

Note : In a parallelogram sum of squares of two diagonals = 2(sum of squares of two adjacent sides)

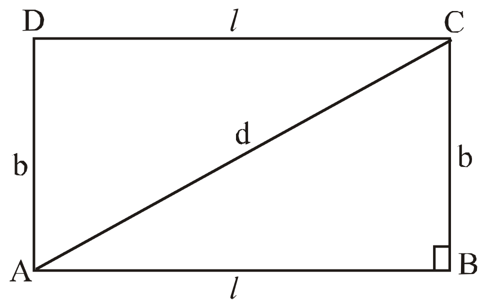
i.e., $d_1^2 + d_2^2 = 2(a^2 + b^2)$ from.



(2) **Rectangle :** A rectangle is a quadrilateral whose opposite sides are equal and all four included angles are 90º.

Note : Diagonals of rectangle are equal and bisect each other.

Length of diagonal = $\sqrt{l^2 + b^2} = d$



Area of rectangle (A) = $l \times b = l \times \sqrt{d^2 - l^2} = b \times \sqrt{d^2 - b^2}$

[If any one side and diagonal is given]

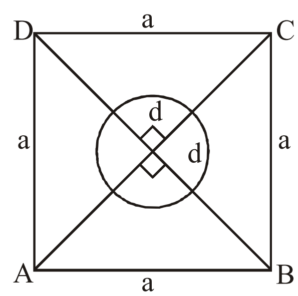
Perimeter of rectangle (P) = $2(l + b)$

$P = 2\left(l + \sqrt{d^2 - l^2}\right)$ [If one side and length of diagonal are given]

(3) **Square :** A square is a quadrilateral with all sides equal and all four included angles equal to 90º.

Note : diagonals of a square are equal and bisect each other at 90º.

Length of diagonal (d) = $a\sqrt{2}$ (by Pythagoras theorem)



$$A = \left(\frac{d}{\sqrt{2}}\right)^2 = \frac{d^2}{2}$$

Perimeter of square (P) = 4 × side = 4 × a = 4a $\Rightarrow a = \frac{P}{4}$

$\therefore$ Area (A) = $\left(\frac{P}{4}\right)^2 = \frac{P^2}{16}$

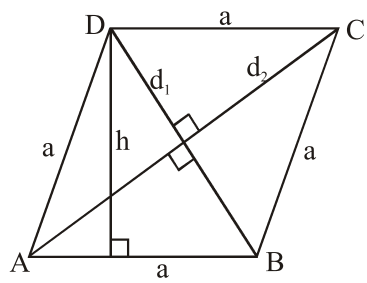
also, $d = a\sqrt{2}$

$d = \frac{P}{4} \times \sqrt{2} \quad \left(\because a = \frac{P}{4}\right)$ $\therefore$ $d = \frac{P}{2\sqrt{2}}$.

(4) Rhombus : A rhombus is a quadrilateral whose all sides are equal.

Note : Diagonals of a rhombus bisect each other at 90º.

In rhombus ABCD.



$$a^2 = \frac{d_1^2}{4} + \frac{d_2^2}{4} \qquad \text{[Pythogoras theorem]}$$

$$\Rightarrow \quad a = \frac{1}{2}\sqrt{d_1^2 + d_2^2}$$

Perimeter of rhombus (P) = $4a = 4 \times \frac{1}{2}\sqrt{d_1^2 + d_2^2} = 2\sqrt{d_1^2 + d_2^2}$

Area of rhombus A = a × h (base × height)

$$A = \frac{1}{2} \times d_1 \times d_2 \text{ i.e., } \left(\frac{1}{2} \times \text{product of diagonals}\right)$$

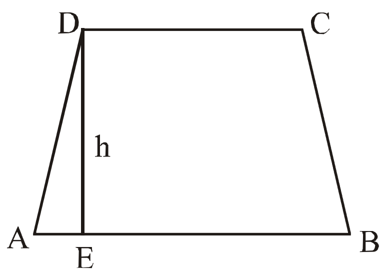
$$A = \frac{1}{2} \times d_1 \times 2\sqrt{a^2 - \left(\frac{d_1}{2}\right)^2}$$

$$A = d_1 \times \sqrt{a^2 - \left(\frac{d_1}{2}\right)^2}$$

(5) Trapezium : A trapezium is a quadrilateral whose any two opposite sides are parallel.

Distance between parallel sides of a trapezium is called height of trapezium.

In fig. ABCD is a trapezium, whose sides AB and CD are parallel, DE = h = height of trapezium = distance between || sides.



Area of trapezium (A) = $\frac{1}{2} \times (AB + CD) \times DE$

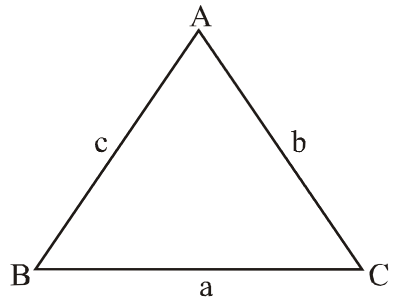
$$= \frac{1}{2} \text{ (sum of || sides)} \times \text{height}$$

Perimeter (P) = (AB + BC + CD + DA)

16.5 HERON'S FORMULA

The formula given by Heron about the area of a triangle is known as Heron's formula.

Area of triangle ABC = $\sqrt{s(s-a)(s-b)(s-c)}$.



where a, b, c are the sides of the triangle, and s is semiperimeter given by $s = \dfrac{a+b+c}{2}$.

16.5.1 Application of Heron's formula :

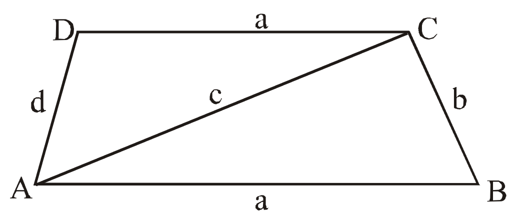
(1) Area of Quadrilateral

We have already discussed how to find area of various quadrilaterals (Parallelogram, Square, Rhombus, Trapezium etc.) Here we will discuss some special cases where Herons' formula can be applied.

Case-I : When four sides of quadrilateral and a diagonal are given.

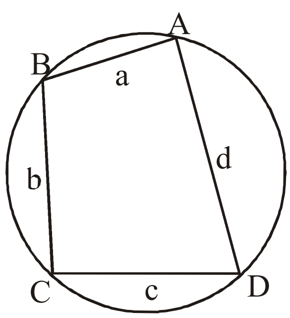
In figure Area of quadrilateral ABCD = Area of ΔABC + Area of ΔACD.

Area of two Δ's ABC and ACD can be found by Heron's formula.



Case-II : Area of a cyclic Quadrilateral

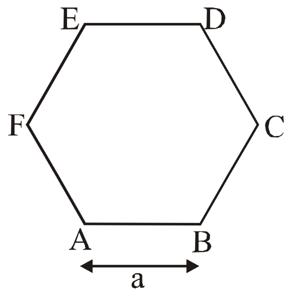
Given quadrilateral ABCD with sides measuring a, b, c, d.



Area = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$ where $s = \dfrac{a+b+c+d}{2}$

(2) Area of Regular Hexagon :

Area = $\frac{3\sqrt{3}}{2}$ a^2, where 'a' is the length of each side of the Regular Hexagon.

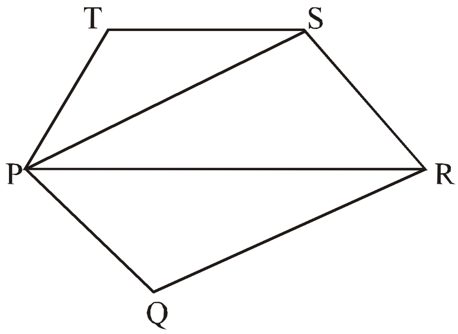


(3) Area of Irregular Plane figures :

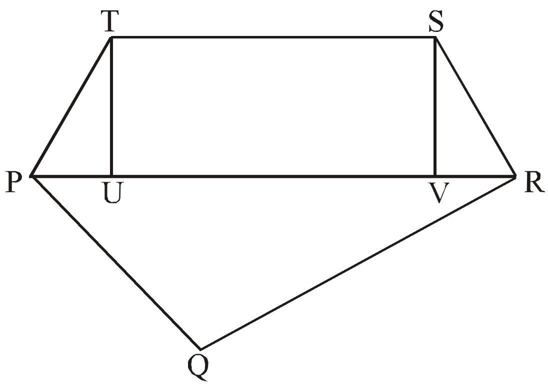
We have already studied what a polygon is, it is a closed figure bounded by three or more straight lines. A polygon with 5 sides is called a pentagon and that with 6 sides is called a hexagon.

We know how to find area of a triangle. We also know how to find area of quadrilateral by spliting it into triangles. Similar methods can be used to find the area of a polygon. Consider following two cases :

(A) By drawing two diagonals PR and PS, the pentagon PQRST is divided into three triangles.



(B) By drawing diagonal PR and two perpendicular TU and SV on this diagonal, Pentagon PQRST is divided into four parts.



This method is known as field.

Area of pentagon PQRST = Area of ΔPTU + Area of trapezium (TUVS) + Area of ΔSVR

= Area of ΔPQR.

16.6 AREA RELATED TO A CIRCLE

16.6.1 Circle

A circle is a path in a plane travelled by a point which moves in such a way that its distance from a fixed point is always constant.

The fixed point is called centre of circle and fixed distance is called radius of the circle.

Circumference or perimeter of circle of radius 'r', is

$c = 2\pi r = \pi d \quad (2r = d, d = \text{diameter})$

Area of circle of radius 'r' $= \pi r^2$

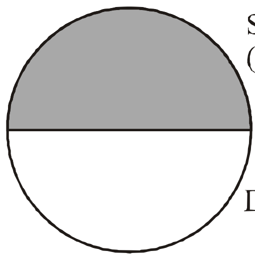
$$= \frac{\pi d^2}{4} = \frac{(\pi d)^2}{4\pi} = \frac{c^2}{4\pi} \qquad [\because c = \text{circumference}]$$

$$= \frac{c^2}{2.2\pi r} \times r$$

Area of circle $= \dfrac{c^2}{2c} \times r = \dfrac{1}{2} \times c \times r$

16.6.2 Semi-circle

A semicircle is a figure enclosed by a diameter and part of circumference of the circle cut off by it.

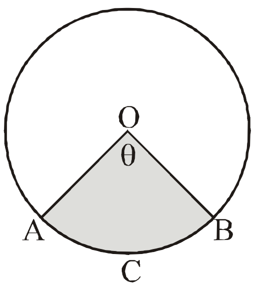


Area of semicircle of radius 'r' $= \dfrac{\pi r^2}{2}$

Circumference of semicircle of radius 'r' $= \pi r + 2r = r(\pi + 2)$

16.6.3 Sector of Circle

Sector is the portion of a circle enclosed by two radii and arc cut by two radii of the circle. OACB is a sector of circle.



Let radius of circle = 'r'

Circumference ($2\pi r$) makes an angle 360º at the centre.

$\therefore$ Length of arc ACB (which makes angle θ at centre) $= 2\pi r \times \dfrac{\theta}{360}$

Area of sector OACB $= \dfrac{1}{2} \times$ length of arc (ACB) $\times$ radius $= \dfrac{1}{2} \times \left(2\pi r \times \dfrac{\theta}{360^\circ}\right) \times r$

Area of sector OACB $= (\pi r^2) \times \dfrac{\theta}{360^\circ}$

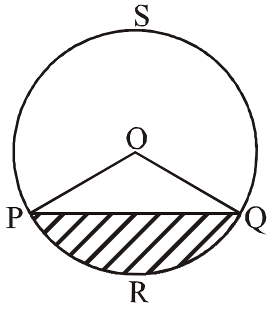
16.6.4 Segment

A segment of a circle is a region enclosed by a chord and an arc which it cuts off by the chord of the circle.

Any chord of a circle which is not a diameter (such as PQ) divides the circle into two segments, one greater (major segment) and one less (minor segment) than a semi-circle.

Area of segment PQR (Minor segment) = Area of sector OPRQ – Area of ΔOPQ

Area of segment PSQ (Major Segment) = Area of Circle – Area of Segment PQR.



16.7 SURFACE AREA AND VOLUME OF SOLIDS

16.7.1 Solid

A solid is a figure which has three dimension namely length, breadth (or width) and height (or thickness). The plane surfaces that bind it are called faces and the lines where faces meet are called edges.

The area of the plane surface that bind the solid is called its surface area.

The amount of space any solid figure occupy in three dimensional space is called its volume.

Let us discuss surface area and volume of some important solids.

(A) **Cuboid :** A cuboid is a three dimensional box. It has six rectangular faces. It is defined by the virtue of its length(ℓ), breadth (b) and height (h). It can be visualised as a room. It is also called rectangular parallelopiped.

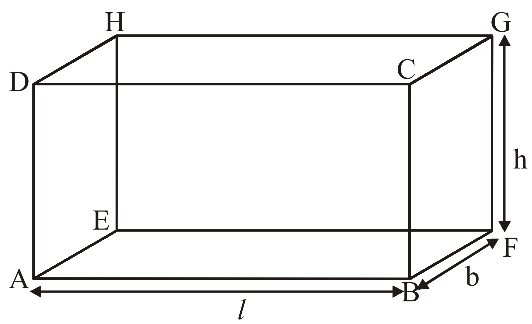


Figure shows a cuboid with length 'l' , breadth 'b' and height 'h'. 'd' denotes the diagonal of the cuboid (AG or CE or BH or DF)

In a cuboid

Area of front face (ABCD) = Area of back face (EFGH)

Area of base (ABFE) = Area of top (DCGH)

Area of side (BCGF) = Area (AEHD)

Let A_F, A_B and A_S denotes area of front, base and side faces respectively.

$\therefore \quad A_F = l \times h,\ A_B = l \times b,\ A_S = b \times h$

Total surface area $= $ sum of area of all faces

$= 2A_F + 2A_B + 2A_S$ Sq. units

$= 2(A_F + A_B + A_S)$ Sq. units

Total surface (TSA) $= 2(lh + lb + bh)$ Sq. units.

Lateral surface area (LSA) is the area of four walls (excluding area of base and top)

$\therefore \quad LSA = 2(A_F + A_S) = 2(lh + bh) = 2h(l + b)$ Sq. Units

Length of diagonal of cuboid $= \sqrt{l^2 + h^2 + b^2} = d$

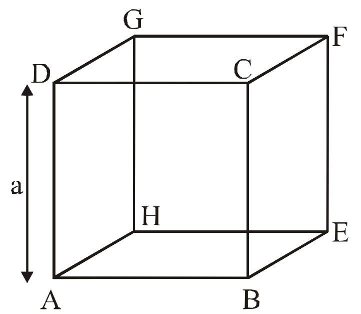
Volume of cuboid = Space occupied by cuboid

= Area of base × height $= (l \times b) \times h$ cubic units $\Rightarrow V = l \times b \times h$

Volume of cuboid $= \sqrt{l^2 \times b^2 \times h^2} = \sqrt{(l \times h) \times (b \times l) \times (b \times h)}$

Volume of cuboid $= \sqrt{A_F \times A_B \times A_S}$

(B) **Cube :** A cube is a cuboid which has all its edges equal i.e.,
length = breadth = height = 'a' say.



Area of each face of the cube is a^2 square units.

Total surface area (TSA) of square = Area of 6 square faces of cube

$TSA = 6 \times a^2 = 6a^2$ sq. units.

Lateral surface area of cube (LSA) = Area of four faces (excluding bottom and top face)

$LSA = 4 \times a^2$

$LSA = 4a^2$ sq. units.

Length of diagonal (d) of cube $= \sqrt{a^2 + a^2 + a^2} = \sqrt{3a^2} = a\sqrt{3}$

Volume of cube (V) = Base area × Height

$V = a^2 \times a = a^3$ cubic units $= \left(\frac{d}{\sqrt{3}}\right)^3$ cubic units

Also, $\quad T.S.A. = 6a^2 \quad (\because d = a\sqrt{3})$

$\therefore \quad TSA = 6 \times \left(\frac{d}{\sqrt{3}}\right)^2 = 2d^2$ sq. units

(C) Cylinder : A right circular cylinder is a solid with circular ends of equal radius and the line joining their centres perpendicular to them. This is called axis of the cylinder. The length of axis (between centres of two circular ends) is called height of the cylinder.

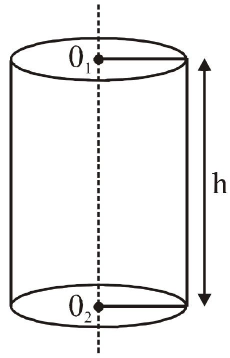


Figure shows a cylinder with 'r' as radius of circular ends and height 'h' (O_1O_2)

Curved surface area of a cylinder (CSA)

= Material required to roll a cylinder (Ignoring thickness)

= circumference of base × height

C.S.A. of cylinder = $2\pi r \times h$

C.S.A. of cylinder = $2\pi rh$ sq. units

If cylinder is closed at both ends then the total surface area of cylinder (TSA)

$$= \text{C.S.A.} + \text{Area of circular ends} = 2\pi rh + 2 \times \pi r^2$$

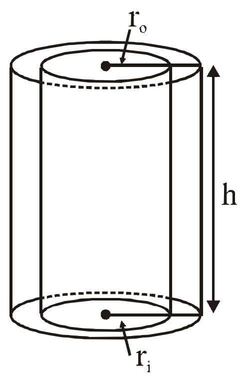
T.S.A. of cylinder = $2\pi r\,(h + r)$ sq. units

Volume of cylinder (V) = Space occupied by it.

= Base area × Height = $\pi r^2 \times h$

Volume of cylinder = $\pi r^2 h$ cubic units.

(D) Right Hollow Cylinder :



A metallic pipe (portion of it shown). If inner radius = r_i and outer radius = r_o then $r_o - r_i$ = thickness of material of pipe.

Pipe is a hollow cylinder. Let its length/height be 'h'

Curved surface area (C.S.A) of hollow cylinder

= C.S.A. of outer cylinder + C.S.A. of inner cylinder

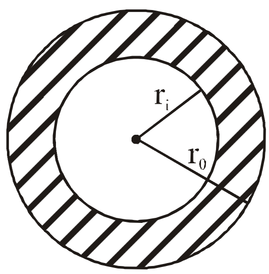
$= 2\pi r_o h + 2\pi r_i h$

C.S.A. of hollow cylinder = $2\pi h\,(r_o + r_i)$ sq. units

T.S.A. of hollow cylinder

= C.S.A. of hollow cylinder + area of 2 circular end rings.

$$= 2\pi h(r_o + r_i) + 2\pi\left(r_o^2 - r_i^2\right)$$



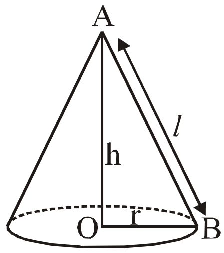
T.S.A. of hollow cylinder = $2\pi(r_o + r_i)\,(h + r_o - r_i)$ sq. units.

Volume of hollow cylinder = Volume of material

Volume of hollow cylinder = $\pi\left(r_o^2 - r_i^2\right)$ h. cubic units.

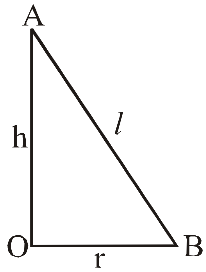
(E) **Right Circular Cone :** A right circular cone is a solid obtained by rotating a right angled triangle around its height. It has a circular base and a slanting lateral surface and coverages at the apex. Its dimensions are defined by the radius of the base (r), the height (h) and slant height (l).

Note : A structure similar to cone is the ice-cream cones.



Height of cone (AO) is always perpendicular to base radius (OB) in a right circular cone.

$\therefore$ From ΔAOB (right angle triangle)



Slant Height (l) = $\sqrt{h^2 + r^2}$

Volume of cone = $\frac{1}{3} \times$ base area $\times$ height = $\frac{1}{3} \times \pi r^2 \times h$

Curved surface area = $\pi r l$ (l = slant height)

Curved surface area (C.S.A) = $\pi r \sqrt{h^2 + r^2}$ sq. units

Total surface area (T.S.A.) = C.S.A. + Base Area = $\pi r l + \pi r^2$

T.S.A. = $\pi r (l + r)$ sq. units

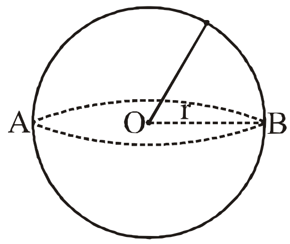
(F) **Sphere :** A sphere is a solid figure formed by revolving a semi circle on its diameter.

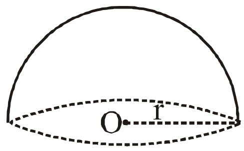
It has one curved surface which is such that all points on it are equidistant from a fixed point within it, called the centre (O).

Any line drawn from the centre to the curved surface is radius (r).

Any line drawn through the centre and terminated at the curved surface is called the diameter (d) of the sphere d = 2r.

A plane through the centre of the sphere cuts the sphere into two equal parts. Each part is called a hemisphere.





Surface area of sphere = $4\pi r^2 = 4\pi \left(\frac{d}{2}\right)^2$.

Surface area of sphere = πd^2

Volume of sphere (V) = $\frac{4}{3}\pi r^3$, $r = \frac{d}{2}$

$\therefore$ $V = \frac{4}{3}\pi \frac{d^3}{8} = \frac{\pi d^3}{6}$

Volume of hemisphere = $\frac{2}{3}\pi r^3 = \frac{\pi d^3}{12}$ $\left(\because r = \frac{d}{2}\right)$

Curved surface area (C.S.A.) of hemisphere = $2\pi r^2$

Total surface area (TSA) of hemisphere = CSA + base area = $2\pi r^2 + \pi r^2$

T.S.A. of hemisphere = $3\pi r^2 = \frac{3}{4}\pi d^2$ $\left(\because r = \frac{d}{2}\right)$

16.8 MEASURES OF VOLUME

1000 cubic millimeters = 1 cubic centrimetre

1000 cubic centrimetre= 1 cubic decimetre

1000 cubic decimeters = 1 cubic metre

or 1000000 cubic centrimetres = 1 cubic metre

1000 cubic meters = 1 cubic decametre

1000 cubic decametres = 1 cubic hectometre

1000 cubic hectometers = 1 cubic kilometre

or 1000000000 cubic metres = 1 cubic kilometre

In order to measure liquids, we use measures of capacity whose units in a litre.

1 litre = 1000 cubic centimetres.

1000 litre = 1cubic metre.

SOLVED EXAMPLE

Example 1 :

A flooring tile has the shape of a parallelogram whose base is 24 cm and the corresponding height is 10cm. How many such tiles are required to cover a floor of area 1080 m^2 ?

Solution :

Length of the base of the tile = 24 cm

And its corresponding height = 10cm

$\therefore$ Area of 1 tile $= \text{base} \times \text{height}$

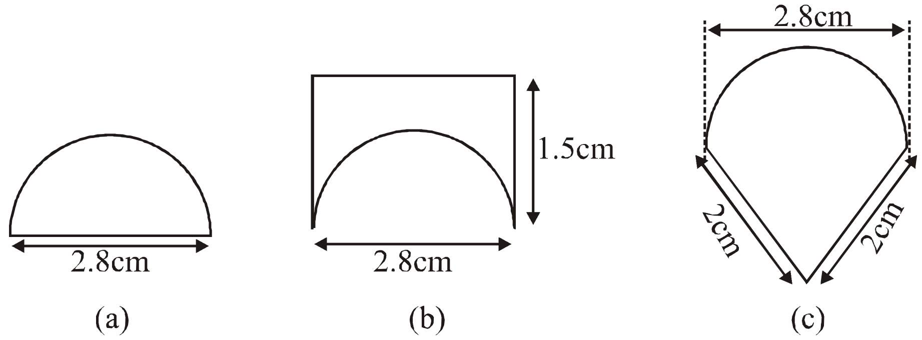
$$= 24 \times 10\text{cm}^2 = \frac{24 \times 10}{100 \times 100}\text{m}^2 = 0.024 \text{ m}^2$$

Floor area = 1080 m^2

$\therefore$ Required number of tiles $= \dfrac{\text{floor area}}{\text{area of 1 tile}} = \dfrac{1080}{0.024} = 45{,}000$

Example 2 :

An ant is moving around a few pieces of different shapes scattered on the floor. For which food piece would the ant have to take a longer round ?



Solution :

Perimeter of figure (a) $= \left[2.8 + \pi \times \frac{2.8}{2}\right]\text{cm} = \left(2.8 + \frac{2.8}{2} \times 3.14\right)\text{cm}$

$= (2.8 + 4.4)\text{cm} = 7.2 \text{ cm}$

Perimeter of figure (b) $= \left[2.8 + 1.5 + 1.5 + \pi \times \frac{2.8}{2}\right]\text{cm} = \left[5.8 + \frac{22}{7} \times 1.4\right]\text{cm}$

$= (5.8 + 4.4)\text{cm} = 10.2 \text{ cm}$

Perimeter of figure (c) $= \left[2 + 2 + \pi \times \frac{2.8}{2}\right]\text{cm} = \left[2 + 2 + \pi \times \frac{2.8}{2}\right]$

$= (4 + 4.4)\text{cm} = 8.4 \text{ cm}$

Hence, for food piece (b), the ant has to take a longer round.

Example 3 :

Area of a trapezium is 720 cm². If the parallel sides are 8 cm and 12 cm long, find the distance between them.

Solution :

Area of the trapezium = 720 cm^2

Parallel sides of the trapezium are 8cm and 12cm.

let the distance between the parallel sides be h cm.

$\because$ Area of the trapezium = $\frac{1}{2}$ × (Sum of the lengths of parallel sides) × (Distance between them)

$\therefore \quad 720 = \frac{1}{2} \times (8 + 12) \times h$

$\Rightarrow \quad 10h = 720$

$\Rightarrow \quad h = \frac{720}{10} = 72$ cm

Distance between the parallel sides = 72 cm.

Example 4 :

A water tank is 1.4m long, 1 m wide and 0.7m deep. How many litres of water can it hold ?

Solution :

Length of the tank (l) = 1.4 m = 140 cm

Breadth of the tank (b) = 1 m = 100 cm

Depth of the tank (h) = 0.7 m = 70 cm

Capacity of the tank = Volume of the tank

$= l \times b \times h$

$= (140 \times 100 \times 70)\text{cm}^3 = \frac{140 \times 100 \times 70}{1000}$ litres

Hence, the tank can hold 980 litres of water.

Example 5 :

A box is 1.8m long, 80 cm wide, 60 cm high. How many soap cakes can be put in it if each cake measures 6 cm by 4.5 cm by 40 mm ?

Solution :

Length of the tank (l) = 1.8 m = 180 cm

Breadth of the tank (b) = 0.8 m = 80 cm

Depth of the tank (h) = 0.6 m = 60 cm

Capacity of the tank = $l \times b \times h$

$= 180 \times 80 \times 60\text{cm}^3 = 864000\ \text{cm}^3$

Length of a soap cake = 6 cm

Breadth of a soap cake = 4.5 cm

Height of a soap cake = 40 mm = 4cm

Volume of one soap cake = $6 \times 4.5 \times 4\text{cm}^3 = 108.0\ \text{cm}^3$

$\therefore$ Required number of soap cakes $= \frac{\text{volume of the box}}{\text{volume of one soap cake}} = \frac{86400}{108} = 8000$

Hence, 8000 soap cakes can be put in the box.

Example 6 :

How many cubes of sides 3 cm can be cut from a solid cuboid whose length, breadth and height are 21 cm, 9 cm and 5 cm respectively.

Solution :

Length of the cuboid (l) = 21 cm

Breadth of the cuboid (b) = 9 cm

Depth of the cuboid (h) = 5 cm

Volume of the cuboid $= l \times b \times h$

$= (21 \times 9 \times 5)$ cm^3

Also, edge of cube = 3 cm

Volume of the cube = (edge)^3

$= (3 \times 3 \times 3)$ cm^3 = 27 cm^3

$\therefore$ Number of cubes which could be obtained from the cuboid $= \dfrac{21 \times 9 \times 5}{3 \times 3 \times 3} = 35$

Example 7 :

A village having a population of 4000, requires 150 *l* water per head per day. It has a tank measuring 20 m by 15 m by 6m. For how many days the water of this tank will last ?

Solution :

Volume of the tank = 20m × 15m × 6m

= 1800 m^3 = 1800000 l

= volume of water consumed by 1 person in 1 day = 150l

$\therefore$ Total volume of water consumed in 1 day = 150 × 4000 l

$\therefore$ Required number of days $= \dfrac{\text{volume of the tank}}{\text{volume of water consumed in 1 day}} = \dfrac{1800000}{150 \times 4000} = 3$

Hence, water of the tank will last for 3 days.

Example 8 :

Find the surface area of a cuboid whose length, breadth and height are 15cm, 12cm and 10cm respectively.

Solution :

Length of the cuboid (l) = 15 cm

Breadth of the cuboid (l) = 12 cm

height of the cuboid (l) = 10 cm

Surface area of a cuboid = 2(lb + bh + hl)

$= 2(15 \times 12 + 12 \times 10 + 10 \times 15)$cm^2

$= 2(180 + 120 + 150)$ cm^2

$= 2(450)$ cm^2

$= 900$ cm^2

Example 9 :

The walls and ceiling of a room are to be painted. If the length, breadth and height of the room are respectively 4.5m, 3m and 3.5m, find the area to be painted.

Solution :

Here, $l = 4.5$m, b =3m, h = 3.5m

Area of four walls of the room $= 2h(l + b)$

$= 2 \times 3.5\ (4.5 + 3)m^2$

$= 7 \times 7.5m^2 = 52.5\ m^2$

Area of the ceiling = area of the floor

$= l \times b = 4.5 \times 3\ m^2 = 13.5\ m^2$

Thus, total area to be painted.

$(52.5 + 13.5)m^2 = 66\ m^2$.

Example 10 :

Ratio of surface area of two cubes is 1 : 4. Find the ratio of their volumes.

Solution :

Let surface area of one of the cubes with edge a_1 be A_1 and surface area of the other cube with edge a_2 be A_2. Let their volumes be V_1 and V_2 respectively.

Then, we have,

$$\frac{A_1}{A_2} = \frac{6a_1^2}{6a_2^2} = \frac{1}{4} \Rightarrow \frac{a_1}{a_2} = \frac{1}{2}$$

$$\therefore \quad \frac{V_1}{V_2} = \frac{(a_1)^3}{(a_2)^3} = \left(\frac{a_1}{a_2}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8} \quad \Rightarrow \quad V_1 : V_2 = 1 : 8.$$

Example 11 :

The radius and height of a cylinder are 14 cm and 51 cm respectively. Find the volume, curved surface area and total surface area of the cylinder.

Solution :

Volume of the cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 14 \times 14 \times 51\ cm^3 = 31416\ cm^3$$

Curved surface area of the cylinder $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 14 \times 51\ cm^2 = 4488\ cm^2$$

Total surface area of the cylinder $= 2\pi r\ (r + h)$

$$= 2 \times \frac{22}{7} \times 14\ (14 + 51)\ cm^3 = 5720\ cm^2.$$

Example 12 :

A cylindrical metallic pole has a radius of 48 cm and height 7 m. Find its volume. If 1m^3 of metal weighs 350 kg, find its weight.

Solution :

Volume of metal used in the pole = $\pi r^2 h$

$$= \frac{22}{7} \times \frac{48}{100} \times \frac{48}{100} \times 7\text{m}^3 = 5.06 \text{ m}^3$$

Weight of the pole = 5.06 × 350 kg = 1771 kg

Example 13 :

A road roller takes 750 complete revolutions to move once over to level a road. Find the area of the road, if the diameter of the road roller is 84 cm and length is 1m.

Solution :

Radius of the road roller = $\frac{84}{2}$ cm = 42 cm

Length of the road roller = 1m = 100 cm

Curved surface area of the roller = area covered by the roller in 1 round = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 42 \times 100 \text{ cm}^2 = 26{,}400 \text{ cm}^2$$

$\therefore$ In 750 revolutions, area covered by the roller

$$= 750 \times 26{,}400 \text{ cm}^2 = \frac{750 \times 26400}{100 \times 100} \text{ m}^2 = 1{,}980\text{m}^2$$

Hence, area of the road = 1980 m^2.

Example 14 :

It costs Rs. 2200 to paint the inner curved surface of a cylindrical vessel 10m deep. If it is painted at the rate of Rs. 20 per sq. m, find the :

(i) inner curved surface area of the vessel.

(ii) radius of the base

(iii) capacity of the vessel

Solution :

Total cost to paint the inner curved surface = Rs. 2200

Rate of painting = Rs. 20 per sq. m

(i) Inner curved surface area of the vessel = $\frac{2200}{20}$ m^2 = 110 m^2

(ii) Height (depth) of the vessel (h) = 10m

Curved surface area of the vessel = $2\pi rh$

$\Rightarrow \quad 110 = 2 \times \frac{22}{7} \times r \times 10 \qquad \Rightarrow \quad r = \frac{110 \times 7}{2 \times 22 \times 10} = 1.75$ m

(iii) Capacity of the vessel = Volume of the vessel

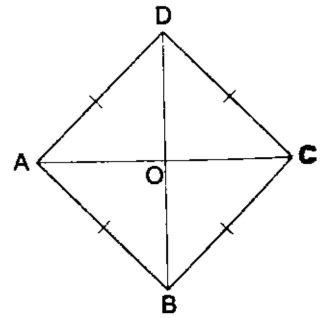
$$= \pi r^2 h = \frac{22}{7} \times 1.75 \times 1.75 \times 10 \text{ m}^3$$

$$= 96.25 \text{ m}^3 = 96.25 \text{ kl}$$

Example 15 :

Perimeter of a rhombus is 146 cm and length of one of its diagonals is 55 cm. Find the length of the other diagonal and area of the rhombus.

Solution :



Perimeter of a rhombus = 146 cm

Each side of the rhombus = $\frac{146}{4}$ cm = 36.5 cm

Since diagonals of a rhombus bisect each other at right angles,

$$AO = \frac{AC}{2} = \frac{55}{2} \text{ cm} = 27.5 \text{ cm}$$

Also ΔAOB is a right angled triangle right angled at O.

$\therefore$ Using Pythagoras theorem, we get,

$(AB)^2 = (AO)^2 + (OB)^2$

or $(36.5)^2 = (27.5)^2 + OB^2$

or $(OB)^2 = (36.5)^2 - (27.5)^2$

$= (36.5 + 27.5)(36.5 - 27.5)$ $[\because x^2 - y^2 = (x+y)(x-y)]$

$= 64.0 \times 9.0 = (8)^2 (3)^2$

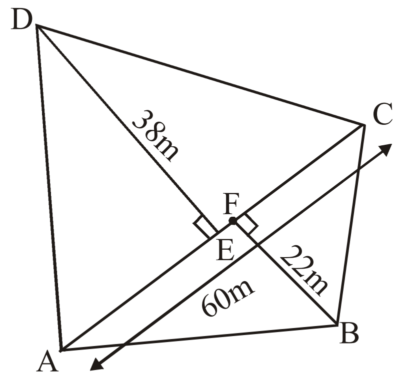
$\therefore$ OB $= 8 \times 3 = 24$

Diagonal (BD) $= 2 \times OB$ cm $= 2 \times 24$ cm $= 48$ cm

$\therefore$ Area of the rhombus $= \frac{1}{2} d_1 \times d_2 = \frac{1}{2}(55 \times 48)\text{cm}^2 = 1320 \text{ cm}^2$

Example 16 :

Find the area of a plot of land which is in the form of a quadrilateral with one diagonal of length 60m and lengths of the perpendiculars drawn from the opposite vertices on this diagonal are 38m and 22m respectively.



Solution :

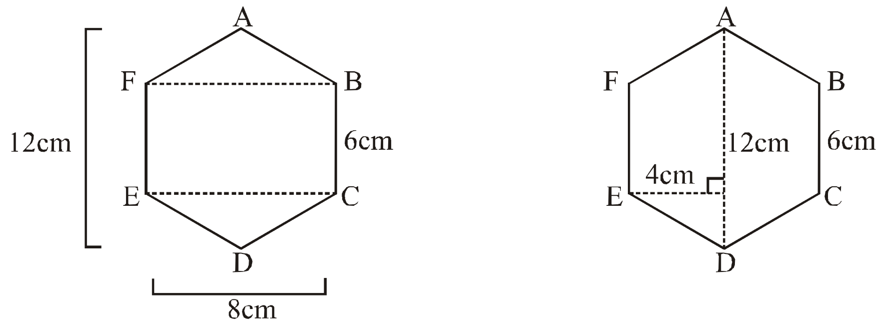
Area of the quadrilateral ABCD = $\frac{1}{2}$ AC(DE + BF)

Required area $= \frac{1}{2} \times 60(38 + 22)\text{m}^2 = 30 \times 60 \text{ m}^2 = 1800 \text{ m}^2$

Example 17 :

There is a regular hexagon ABCDEF of side 6 cm. It is divided in two ways. Find the area of hexagon using both the methods.

Solution :



(a) (b)

According to figure (a), ΔABF and ΔCDE are congruent triangles with altitude 3 cm.

Area of $\Delta ABF = \frac{1}{2} \times 8 \times 3 = 12 \text{ cm}^2$

Area of ΔDCE

Area of Rectangle BCEF = $8 \times 6 = 48 \text{ cm}^2$

Area of hexagon ABCDEF = $12 + 12 + 48 = 72 \text{ cm}^2$

According to figure (b), AD divides hexagon ABCDEF into two congruent trapeziums.

Area of trapezium ABCD = $\frac{1}{2} \times (12 + 6) \times 4 = 36 \text{ cm}^2$

Area of hexagon ABCDEF = Area of trapezium ABCD + Area of trapezium ADEF

$= 36 + 36 \text{ cm} = 72 \text{ cm}^2$

Example 18 :

The edge of a cube is 15 metre. Find its(a) lateral surface area, (b) whole surface area.

Solution :

(a) We know that side of cube = 15m

Lateral surface area of cube = $4a^2$ square units = $(4 \times 15 \times 15)\text{m}^2 = 900 \text{ m}^2$

(b) Whole surface area $= 6a^2 = (6 \times 15 \times 15)\text{m}^2 = 1350 \text{ m}^2$

Example 19 :

A rectangular room measuring 3.4m × 2.5m × 2.6m. The walls of the room are painted. Find the cost of painting the four walls of the room at the cost of Rs. 22 per m^2.

Solution :

Length l of the room = 3.4m

Breadth b of the room = 2.5m

Height h of the room = 2.6m

Area of four walls of the room = 2(length + breadth) × height

$= 2(3.4 + 2.5)\ 2.6 = 2(5.9)\ (2.6) = 30.68 \text{ m}^2$

Cost of painting 1m^2 = Rs. 22

Cost of painting 30.68 = Rs. 22 × 30.68 = Rs. 674.96

Cost of painting the room is Rs. 674.96

Example 20 :

A roller is 210 cm long having the diameter as 35 cm. It takes 500 complete revolutions to level a playground. Find the area of the playground.

Solution :

Height, h of the roller = 210 cm

diameter of the roller = 35 cm

radius r of the roller = $\frac{35}{2}$ cm

Area of the playground levelled in 1 revolution = lateral surface area of the roller

$$= 2\pi rh = 2 \times \frac{22}{7} \times \frac{35}{2} \times 210 \text{ cm}^2 = 23100 \text{ cm}^2$$

Area of the playground = Area of the playground covered in 1 revolution × 500

$= 23100 \times 500 \text{ cm}^2 = 11550000 \text{ cm}^2 = 1155 \text{ m}^2$

Area of the playground is 1155 m^2.

Example 21 :

There are 20 cylindrical pillars in a path. The radius of each pillar is 14 cm and height is 1m. Find the total cost of painting the curved surface area of all the pillars the rate of Rs. 10 per square metre.

Solution :

Radius, r of the pillar = 14 cm = 0.14 m

Height, h of the pillar = 1m

Curved surface area of 1 pillar $= 2prh = 2 \times \frac{22}{7} \times 1.4 \times 1 = 0.88 \text{ m}^2$

Curved surface area of 20 pillars = 0.88×20 m = 17.6 m^2

Cost of painting 1m^2 = Rs. 10

Cost of painting 17.6 m^2 = Rs. 10 × 17.6 = Rs. 176

Thus, total cost of painting is Rs. 176

Example 22 :

volume of a cube is same as that of a cuboid of dimensions 16m × 8m × 4m. Find the edge of the cube.

Solution :

Volume of the cuboid = $(16 \times 8 \times 4)\text{m}^3 = 512 \text{ m}^3$

Let a be the edge of the cube then

Volume of the cube = $a^3 = 512 \text{ m}^3$ (Given)

or $a^3 = 512$

$\therefore$ $a = \sqrt[3]{512} = 8$

Hence, edge of the cube is 8 m.

Example 23 :

A swimming pool in the shape of cuboid is $37\frac{1}{3}$ m long, 12 m wide and 8m deep. Find the quantity of water (in kl) in the swimming pool.

Solution :

Volume of water in the tank = $\left(37\frac{1}{3}\times12\times8\right)m^3 = \left(\frac{112}{3}\times12\times8\right)m^3$

$= 3584\ m^3 = 3584\ kl$ (as $1m^3 = 1000\ l = 1\ kl$)

Example 24 :

A cuboid having the dimensions as 12 m × 6m × 3m is melted to form a cube. Find the length of each side of the cube.

Solution :

Volume of the cuboid = 12m × 6m × 3m = 216 m^3

A cuboid is melted to form a cube. Thus volume of the cuboid = Volume of the cube.

216 m^3 = $Side^3$

Thus side of the cube = $\sqrt[3]{216}$ m = 6m

Side of the cube is 6m.

Example 25 :

It is required to make a cylindrical can which may hold 1 litre of water.

(a) If its base diameter is 10 cm, what must be the height of the can ?

(b) If the height is $3\frac{2}{11}$ cm, what must be the diameter of the can ?

Solution :

(a) Let height of the can be h cm

$r = \frac{1}{2} \times 10cm = 5cm$ ($\because$ Diameter = 10 cm given)

Then, $V = \pi r^2 h$ cu units $= \frac{22}{7} \times 5 \times 5 \times h\ cm^3$ .... (i)

Volume of the can holding 1 litre water = 1000 cm^3 .... (ii)

$\therefore \quad = \frac{22}{7} \times 5 \times 5 \times h = 1000$ [Using (i) and (ii)]

$h = \frac{1000\times7}{22\times5\times5} = 12\frac{8}{11}$ cm

(b) Let radius be r cm

$h = 3\frac{2}{11}\text{ cm} = \frac{35}{11}\text{ cm}$

$\therefore \quad V = \left(\frac{22}{7} \times r^2 \times \frac{35}{11}\right)$

$\therefore \quad \frac{22}{7} \times r^2 \times \frac{35}{11} = 1000$ ... [Using (ii)]

or $\quad r^2 = \frac{1000 \times 7 \times 11}{22 \times 35} = 100 = (10)^2$

$\therefore \quad r = 10\text{ cm}$

Hence, required diameter, $2 \times 10 = 20$ cm

Example 26 :

In modern building a reinforced concrete column is cylindrical in shape, having a diameter of 1 metre and height of 21 metres. What would be the cost of concrete for the column at Rs. 260 per cubic metre ?

Solution :

Here, $r = \frac{1}{2}$ m

As per given condition, volume of concrete required

$= \left(\frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times 21\right)\text{m}^3 = \frac{33}{2}\text{ m}^3$

Cost of 1m^3 of concrete = Rs. 260

$\therefore$ Cost of $\frac{33}{2}\text{ m}^3$ of concrete = $\frac{33}{2} \times$ Rs. 260

Thus, total cost = Rs. 4290

Example 27 :

A rectangular piece of paper 22 cm × 12 cm is folded to make a cylinder of height 12 cm. Find the volume of cylinder. Assume there is no overlapping of paper.

Solution :

Length of paper becomes the perimeter of the base of the cylinder and width becomes height.

Let radius of the cylinder = r and height = h.

Perimeter of the base of the cylinder = $2\pi r = 22 = 2 \times \frac{22}{7} \times r = 22$

$$r = \frac{7}{2}\text{ cm}$$

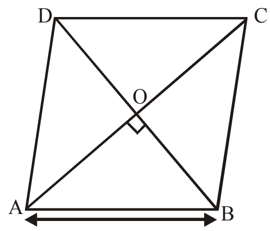
Volume of the cylinder, $V = \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 12\text{cm}^3 = 462\text{ cm}^3$

Example 28 :

Find the area of a rhombus whose each side is 13m and one of the diagonals is 10m.

Solution :

Let ABCD be a rhombus whose each side is 13m. Let the diagonals AC and BD intersect each other at O(see figure). Let BD measure 10m.



Since the diagonals of a rhombus bisect each other at right angles,

$\therefore$ $\angle AOB$ is a right angle.

Now in right ΔAOB, by Pythagora's theorem

$AO^2 + BO^2 = AB^2$

$AO^2 + 5^2 = 13^2$

$AO^2 + 25 = 169$

$AO^2 = 169 - 25 = 144$

$AO = 12$ m

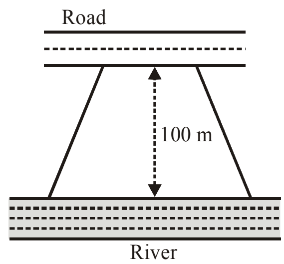
$\therefore$ $AC = 2 \times AO = 2 \times 12$ or 24m

Area of rhombus ABCD = $\frac{1}{2} \times$ product of diagonals

$= \frac{1}{2} \times 24 \times 10$ or 120 m^2.

Example 29 :

Mohan wants to buy a trapezium shaped field. Its side along the river is parallel to and twice the side along the road. If the area of this field is 10500 m^2 and the perpendicular distance between the parallel sides is 100m, find the length of the side along the river.



Solution :

Suppose the length of the side along the road = x m

$\therefore$ Length of the side along the river = 2x m

Sum of parallel sides = x + 2x or 3x m

Area = 10500 m^2

Height = 100m

$\therefore$ $\frac{1}{2} \times 3x \times 100 = 10500$

$x = \frac{10500 \times 2}{3 \times 100} = 70$

Hence length of side along the river = 2x = 2 × 70 or 140m.

Example 30 :

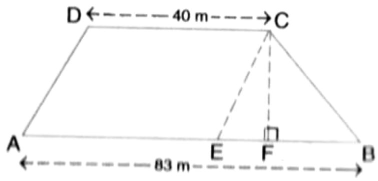
A field ABCD is in the form of a trapezium in which AB || CD, AB = 83m and CD = 40m. A triangular flower bed EBC is cut in such a way that the shape of the remaining field becomes a parallelogram. If the area of the entire field is 2337 m², find the area of (a) flower bed (b) remaining field.

Solution :

From C draw CE || AD and CF ⊥ AB

Now AECD is a parallelogram and EB = AB – AE = 83 – 40 or 43 m.

Area of trap. ABCD = 2337 m^2



$\therefore \quad \frac{1}{2}(83 + 40) \times CF = 2337$

$\therefore \quad CF = \frac{2337 \times 2}{123} m = 38m$

(a) Area of $\Delta EBC = \frac{1}{2} \times EB \times CF = \frac{1}{2} \times 43 \times 38$ m2 = 817 m^2

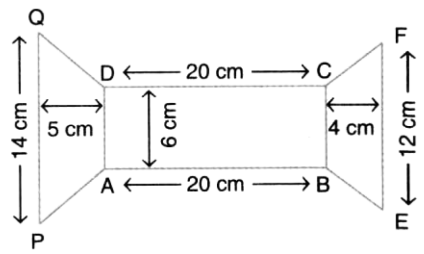
$\therefore$ Area of flower bed = 817 m^2.

(b) Area of remaining field = 2337 m^2 – 817 m^2

= 1520 m^2

Example 31 :

Find the area of the figure drawn.



Solution :

Area of rectangle ABCD

= Length × breadth

= 20 × 6 or 120 cm^2 ..... (i)

Area of trapezium BEFC = $\frac{1}{2}$ (sum of parallel sides) × height

$= \frac{1}{2}(6 + 12) \times 4 \text{ cm}^2 = 36\text{cm}^2$ .....(ii)

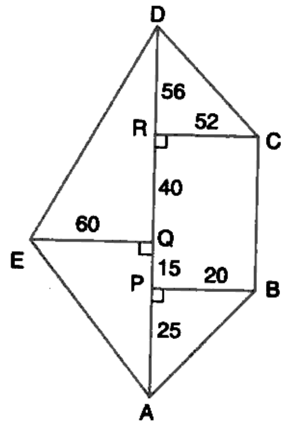
Area of trapezium ADQP = $\frac{1}{2}$ (sum of parallel sides) × height

$= \frac{1}{2}(6 + 14) \times 5 \text{ cm}^2 = 50\text{cm}^2$ .....(iii)

Area of the given figure = 120 cm^2 + 36 cm^2 + 50 cm^2 = 206cm^2

Example 32 :

In figure, the dimensions are given in metres. Find the area of this field.



Solution :

Area of $\Delta APB = \frac{1}{2}$ AP × PB

$= \frac{1}{2} \times 25 \times 20$ m^2

$= 250$ m^2 ... (i)

Area of $\Delta AQE = \frac{1}{2}$ AQ × EQ

$= \frac{1}{2} \times 40 \times 60$ m^2

$= 1200$ m^2 ... (ii)

Area of $\Delta EQD = \frac{1}{2}$ QD × EQ

$= \frac{1}{2} \times 96 \times 60$ m^2

$= 2880$ m^2 ... (iii)

Area of PBCR $= \frac{1}{2}$ (PB + RC) × PR

$= \frac{1}{2}(20 + 52) \times 55$ m^2 $= 1980$ m^2 ... (iv)

Area of $\Delta RCD = \frac{1}{2}$ RD × RC

$= \frac{1}{2} \times 56 \times 52$ m^2

$= 1456$ m^2 ... (v)

Adding (i) to (v), we get

Area of polygon ABCDE = (250 + 1200 + 2880 + 1980 + 1456)m^2 = 7766 m^2

Example 33 :

Top surface of a raised platform is in the shape of a regular octagon whose each side is 5m. Find the area of the platform.

Solution :

Side of the regular octagon = 5m

$\therefore$ Area of the regular octagon $= 2(1 + \sqrt{2})a^2$ ... (a is side of regular octgon)

$= 2(1 + \sqrt{2}) \times 5^2$ sq m

$= 2(1 + 1.4) \times 25$ sq m

$= 2 \times 2.4 \times 25$ sq m

$= 120$ sq m

Example 34 :

Calculate the volume of water in a tank whose base measures 75 cm × 60 cm when the height of the water in the tank is 46 cm. Give your answer in cubic centimetres and also in litres.

Solution :

Length of the tank = 75 cm

Width of the tank = 60 cm

Height of water = 75 cm

Volume of water = length × breadth height

= $75 \times 60 \times 46$ cm^3

= 207000 cm^3

= 207000 ÷ 1000 or 207 litres.

Example 35 :

How many cubes of side 15 cm can be fitted into a box which measures 1.5 m × 90 cm × 75 cm?

Solution :

Side of cube = 15 cm

Volume of cube = $15 \times 15 \times 15$ cm^3 = 3375 cm^3

Length of the box = 1.5 cm = 150 cm

Breadth of the box = 90 cm

Height of the box = 75 cm

Volume of box = $150 \times 90 \times 75$ cm^3 = 1012500 cm^3

$\therefore$ Number of cubes that can fit in the box = $\dfrac{1012500}{3375}$

or $\dfrac{150 \times 90 \times 75}{15 \times 15 \times 15} = 300$

Example 36 :

A cuboid is made of metal. It is 27 cm × 18 cm × 12 cm. It is melted and recast into small cubes with an edge 6 cm in length. How many cubes are made, assuming that there is no wastage in the process ?

Solution :

Volume of metal = $27 \times 18 \times 12$ cm^3 = 5832 cm^3

Volume of the cube = $6 \times 6 \times 6$ or 216 cm^3

Number of cubes = $\dfrac{5832}{216} = 27$ or $\dfrac{27 \times 18 \times 12}{6 \times 6 \times 6} = 27$

Example 37 :

A cube has volume 8000 cm^3. Find its edge.

Solution :

Volume of the cube = 8000 cm^3 = $(20)^3$ cm^3

$\therefore$ Edge of the cube = 20 cm

Example 38 :

A cube of side 12 cm is melted down and reshaped into a cuboidal block of width 15 cm and length 18 cm. How high is the block ?

Solution :

Side of the cube = 12 cm

Volume of the cube = 12 × 12 × 12 or 1728 cm^3

Length of block = 18 cm

Breadth of block = 15 cm

Height of block = $\frac{\text{Volume}}{\text{length} \times \text{breadth}} = \frac{1728}{18 \times 15}$ cm = 6.4 cm

Example 39 :

Find the side of a cube whose surface area is 600 cm^2.

Solution :

Surface area of a cube = 6 $(\text{side})^2$

$\therefore$ $6(\text{side})^2 = 600$

or $(\text{Side})^2 = \frac{600}{6} = 100 = (10)^2$

$\therefore$ Side = 10

Hence side of the cube = 10 cm

Example 40 :

Daniel is painting the walls and ceiling of a cuboidal hall with length, breadth and height 15m, 10m and 7m respectively. From each can paint 100 m^2 of area is painted. How many cans of paint will be required to paint the room ? Find the cost of paint of each can costs Rs. 238.

Solution :

Length of the room = 15 m

Width of the room = 10 m

height of the room = 7 m

Area of 4 walls = 2(length + breadth) × height

= 2(15 + 10) × 7m^2 = 350 m^2

Area of ceiling = length × breadth

= 150 × 10 or 150 m^2

Total area to be painted = (350 + 150) m^2 = 500 m^2

No. of cans required = $\frac{500}{100} = 5$

Cost of 1 can = Rs. 238

Cost of 5 cans = Rs. 238 × 5 = Rs. 1190.

Example 41 :

A well is dug 16m deep. Its radius is 1.75 m. The earth dug out is spread evenly on a rectangular platform which is 11m × 4m. Find the height of the platform raised.

Solution :

Radius of the well = 1.75 m = $\frac{7}{4}$ m

Depth of the well = 16 m

Volume of earth dug out = $\frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 16 \text{ m}^3 = 154 \text{ m}^3$

Now area of the base of the platform = 11m × 4m = 44m^2

$\therefore$ Height of the platform raised = $\frac{154}{44}$ m = $\frac{7}{2}$ or $3\frac{1}{2}$ m

Example 42 :

The rainfall recorded on 21 July was 10 cm. The rain water that fell on a roof 70m long and 44m wide was collected in a cylindrical tank of radius 14m. Find

(a) volume of rain water fell on the roof

(b) rise of water level in the tank due to rain water.

Solution :

(a) Volume of rain water fell on the roof = $70 \times 44 \times \frac{10}{100} \text{ m}^3$

$= 308 \text{ m}^3$

(b) Volume of rain water collected in tank = 308 m^3

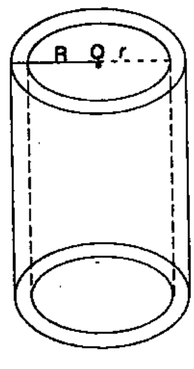
Radius of the base of the tank = 14m

Height of water raised = $\frac{\text{Volume}}{\pi r^2} = \frac{308 \times 7}{22 \times 14 \times 14}$ m = $\frac{1}{2}$ m = 0.5m

Example 43 :

A cylinder is open at ends. The external diameter is 10 cm and thickness 1cm. If the height is 8cm, find the volume of the metal used in the cylinder, (Take π = 3.14)

Solution :



Let R and r be the external and internal radii of the cylinder.

R = 5cm.

r = 5 – 1 or 4 cm

Volume of the outer cylinder = $\pi R^2 h$.

$= 3.14 \times 5 \times 5 \times 8 \text{ cm}^3 = 628 \text{ cm}^3$

Volume of the inner cylinder = $\pi r^2 h$

$= 3.14 \times 4 \times 4 \times 8 \text{ cm}^3$

$\therefore$ Volume of the metal used = volume of outer cylinder – Volume of inner cylinder

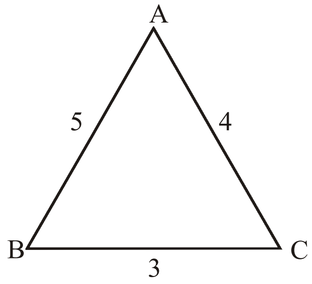
$= (628 - 401.92) \text{ cm}^3 = 226.08 \text{ cm}^3$

Example 44 :

What is the area of a triangle whose sides are 3m, 4m and 5m long ?

Solution :

Let BC = a = 3m, AC = b = 4m, AB = c = 5m



Semiperimeter (s) = $\frac{a+b+c}{2} = \frac{3+4+5}{2} = 6$m

Area $= \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{6(6-3)(6-4)(6-5)}$

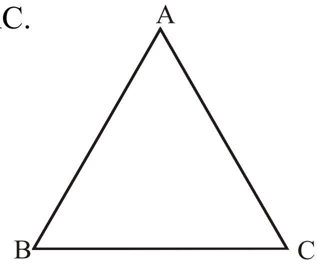
$= \sqrt{6\times3\times2\times1} = \sqrt{36} = 6m^2$.

Example 45 :

The perimeter of an isosceles triangle is 42 cm. If the base is 16 cm, find the length of equal sides.

Solution :

Let ABC be an isosceles D with BC as base and equal sides AB and AC.



Let equal sides AB = AC = 'x' cm

BC = 16 cm (given)

$\therefore$ Perimeter of ΔABC = AB + BC + CA (sum of all sides)

42 = x + 16 + x

$\therefore$ 2x = 26

x = 13

$\therefore$ Equal sides are each 13 cm long.

Example 46 :

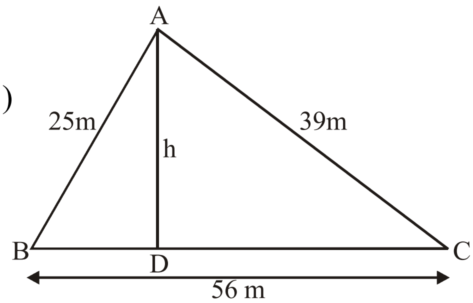
The sides of a triangle are 25m, 39m and 56 m respectively. Find the perpendicular from the opposite vertex to the greatest side.

Solution :

Let BC = a = 56 m, AC = b = 39m, AB = c = 25m.

Let h = length of perpendicular

From vertex A to side BC



Area of ΔABC = $\frac{1}{2}\times BC\times BD = \frac{1}{2}\times 56\times h = 28h$ .....(1)

Also area ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$, s = $\frac{a+b+c}{2}$

$= \frac{59+30+25}{2} = \frac{120}{2} = 60$ m

$\therefore$ Area ΔABC $= \sqrt{60(60-56)(60-39)(60-25)} = \sqrt{60\times4\times21\times35}$

$= \sqrt{3\times5\times4\times4\times7\times3\times7\times5} = \sqrt{3\times3\times4\times4\times5\times5\times7\times7}$

Area ΔABC = 3 × 4 × 5 × 7 = 420 m^2 ..... (2)

From (1) and (2)

28h = 420

h = $\frac{420}{28}$ = 15m

$\therefore$ Height (perpendicular) AD = 15m.

Example 47 :

Sides of a triangle are in the proportion of 4 : 5 : 6 and the perimeter is 195m. Find its area.

Solution :

Let the ratio of sides be 'x'

$\therefore$ Sides are 4x, 5x and 6x. ($\because$ perimeter = 195 cm)

Now $4x + 5x + 6x = 195$

$15x = 195$

$\Rightarrow$ $x = 13$

$\therefore$ Sides are $4 \times 13, 5 \times 13, 6 \times 13$

i.e,. 52m, 65m, 78m

Semi-perimeter(s) $= \frac{195}{2} = 97.5$ m

$\therefore$ Area $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{97.5(97.5-52)(97.5-65)(97.5-78)} = \sqrt{97.5 \times 45.5 \times 32.5 \times 19.5}$

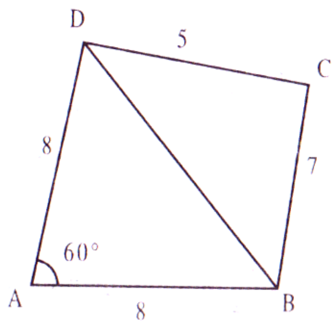
$= \sqrt{2811473.44} = 1676.745 \text{ m}^2$

Example 48 :

The sides of a quadrilateral taken in order are 8, 8, 7 and 5m respectively and the angle contained by the first two sides is 60º, find its area.

Solution :

Given quadrilateral is shown as ABCD where AB = DA = 8m. $\angle DAB = 60^\circ$ as AB = DA.



$\Rightarrow$ $\angle ABD = \angle DBA = 60^\circ$

All angles of DABD are 60º, so it is an equilateral

$\therefore$ Side DB = 8 m

$\therefore$ Area of ABCD = Area ABD + Area BCD

Area ABD $= \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4} \times 82 = 16\sqrt{3} \text{ m}^2$

Area BCD $= \sqrt{s(s-BC)(s-CD)(s-DB)}$, $s = \frac{BC+CD+DB}{2} = \frac{7+5+8}{2} = 10\text{m}$

$\therefore$ Area BCD $= \sqrt{10(10-7)(10-5)(10-8)} = \sqrt{10 \times 3 \times 5 \times 2} = 10\sqrt{3} \text{ m}^2$

$\therefore$ Area ABCD $= 16\sqrt{3} + 10\sqrt{3} = 26\sqrt{3} \text{ m}^2 = 26 \times 1.732 = 45 \text{ m}^2$ (approx)

Example 49 :

The cross section of a canal is trapezium in shape. If the canal is 10m wide at the top and 5m wide at he bottom and area of cross section is 800 m^2, find the depth of the canal.

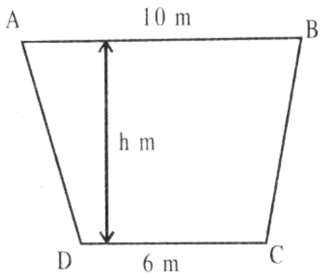
Solution :

Let height of trapezium cross section ABCD = h 'm'

$$\text{Area of trapezium} = \frac{1}{2} \times (AB + CD) \times h\ m^2,$$

$$800 = \frac{1}{2} \times (10 + 6) \times h$$

$$\Rightarrow \quad h = \frac{800 \times 2}{16}\ m = 100m$$



Example 50 :

The area of a parallelogram is 338 m^2. It its altitude is twice the corresponding base, determine the base and the altitude.

Solution :

Let base of parallelogram = 'a' m

$\therefore$ Corresponding altitude = '2a'm

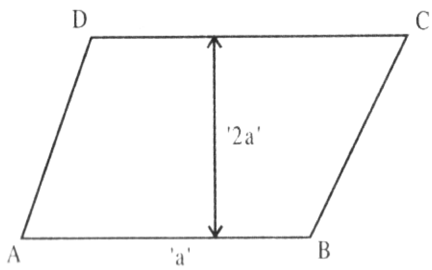
Area of parallelogram ABCD = AB × corresponding altitude

$338 = a \times 2a$

$\Rightarrow \quad a^2 = 169$

$\Rightarrow \quad a = 13m$

Corresponding altitude of paralleogram = 13 × 2 = 26m



Example 51 :

The length of the floor of a rectangular hall is 10m more than its breadth. If 34 carpets of size 6m × 4m are required to cover the floor of the hall, then find the length and breadth of the hall.

Solution :

Let length and breadth of rectangular hall be l and b. It is given $l = (b + 10)$

Area of 34 carpets = Area of floor of hall

$l + b = 10$

$34 \times 6 \times 4 = b \times (b + 10)$

$\Rightarrow \quad b \times (b + 10) = 24 \times (24 + 10 \quad [\because\ 34 = 24 + 10])$

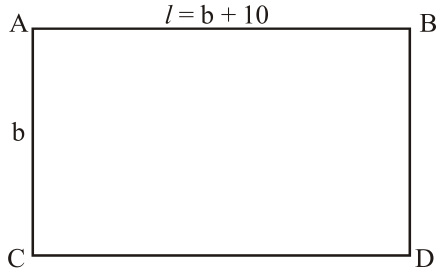
From above b = 24m

and $l = b + 10$

$l = 24 + 10 = 34m$

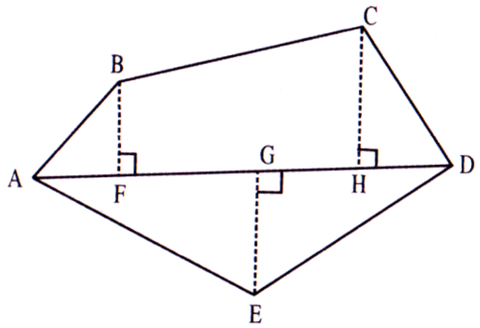
$\therefore$ length of rectangle hall = 34 m

Breadth of rectangle hall = 24 m



Example 52 :

Find the area of the pentagon ABCDE shown below if AD = 8cm, AH = 6 cm, AG = 4cm, AF = 3cm, BF = 2cm, CH = 3cm, and ET = 2.5 cm.



Solution :

Area of pentagon ABCDE = Area of ΔAFB + Area of trapezium BCHF + Area of ΔCHD + Area of ΔAED.

$$= \frac{1}{2} \times (AF \times BF) + \frac{1}{2}(BF + CH) \times (AH - AF) + \frac{1}{2} CH \times (AD - AH) + \frac{1}{2}(AD \times EG)$$

$$= \frac{1}{2}(3 \times 2) + \frac{1}{2}(2 + 3) \times (6 - 3) + \frac{1}{2} \times 3 \times (8 - 6) + \frac{1}{2}(8 \times 2.5)$$

$$= 3 + \frac{15}{2} + 3 + 10 = \frac{6+15+6+20}{2} = \frac{47}{2} = 23.5 \text{ cm}^2$$

∴ Area of pentagon = 23.5 cm^2

Example 53 :

A circular plot covers an area of 154 m^2. How much wire is required for fencing the plot ?

Solution :

Let radius of circular plot = 'r' m

∴ Area of circular plot = πr^2 = 154 (Given)

$$r^2 = \frac{154 \times 7}{22} = 49 \quad \Rightarrow \quad r = 7\text{m}$$

∴ Length of wire required for fencing = circumference of circular plot = $2\pi r = 2 \times \frac{22}{7} \times 7 = 44$m

Example 54 :

Find the length of a rope by which a buffalo must be tethered in order that she may be able to graze an area of 9856 m^2.

Solution :

Let the length of rope be 'l' m.

This length will act as radius of circle in which the buffalo move to graze.

∴ Area of circle (where buffalo is able to graze) = πl^2

$$9856 = \frac{22}{7} \times l^2 \quad \Rightarrow \quad l = \sqrt{\frac{9856 \times 7}{22}}$$

$$l = \sqrt{448 \times 7} = \sqrt{64 \times 7 \times 7} = \sqrt{(8)^2 \times (7)^2} = 7 \times 8 = 56\text{m}.$$

Example 55 :

Find the area of a sector whose radius is 10 cm and the length of the arc is 13 cm.

Solution :

Length of arc which make angle 'θ' at the centre $= 2\pi r \times \dfrac{\theta}{360^o}$

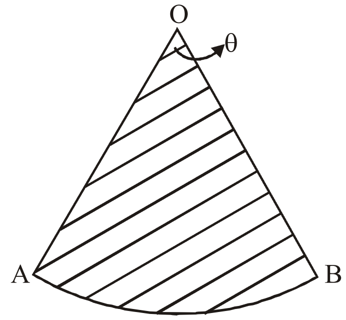
r = radius of the circle

Area of sector made by above arc

$= \pi r^2 \times \dfrac{\theta}{360^o}$

$= \dfrac{1}{2}\left(2\pi r \times \dfrac{\theta}{360^o}\right) \times r$

$= \dfrac{1}{2} \times (\text{length of arc}) \times r = \dfrac{1}{2} \times 13 \times 10 = 65\text{cm}^2$



Example 56 :

How many revolutions will a wheel make in travelling 528m if its diameter measures 0.7m?

Solution :

Let the no. of revolutions made by wheel = 'n'

Distance travelled in one revolution = circumference of wheel

$= \pi \times d$ (d = diameter of wheel)

$= \dfrac{22}{7} \times 0.7 = 2.2\text{m}$

Now, No. of revolutions × Distance travelled in one revolution = Total distance travelled

$\Rightarrow$ No. of revolution $= \dfrac{\text{total distance travelled}}{\text{Distance travelled in one revolution}} = \dfrac{528}{2.2} \times 10 = 240$ revolutions

Example 57 :

The surface area of a cube is 216 sq. cm. Find its volume.

Solution :

Let each side of cube be 'a'

$\therefore$ Surface area of cube $= 6a^2$

$6a^2 = 216 \quad \Rightarrow \quad a^2 = 36 \quad \Rightarrow \quad a = 6$ cm

Volume of cube $= a^3\ \text{cm}^3 = (6)^3 = 216\ \text{cm}^3$

Example 58 :

On a rainy day 60 mm of rain falls, find how many cubic metres of water falls on 3 hectare of ground on that day.

Solution :

Height (h) of the rain water fall = 60 mm $= \dfrac{60}{1000}$ m

Area of ground (A) = 3 hectares $= 3 \times 10000\ \text{m}^2$ (1 hectare $= 10000\ \text{m}^2$)

$\therefore$ Volume of water fall = Area of ground × height $= 3 \times 10000 \times \dfrac{60}{1000} = 30 \times 60 = 1800\ \text{m}^3$

Example 59 :

Volume of two cubes are in the ratio 1 : 27, find the ratio of their surface areas.

Solution :

Let sides of two cubes be a_1 and a_2 respectively.

Volume of first cube $(V_1) = a_1^3$

Volume of second cube $(V_2) = a_2^3$

$$\therefore \quad \frac{V_1}{V_2} = \frac{a_1^3}{a_2^3} = \left(\frac{a_1}{a_2}\right)^3 = \frac{1}{27}$$

$$\therefore \quad \frac{a_1}{a_2} = \frac{1}{3}$$

Now, surface area of first cube $(S_1) = 6a_1^2$

surface area of first cube $(S_2) = 6a_2^2$

$$\text{Now,} \quad \frac{S_1}{S_2} = \frac{6a_1^2}{6a_2^2} = \left(\frac{a_1}{a_2}\right)^2$$

$$\frac{S_1}{S_2} = \left(\frac{1}{3}\right)^2$$

$$\frac{S_1}{S_2} = \frac{1}{9}$$

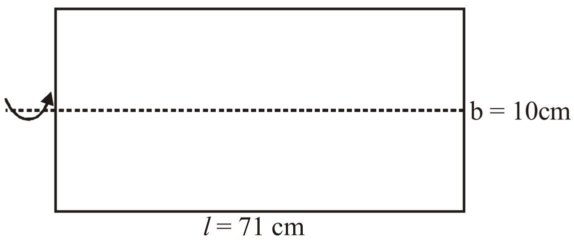
$\therefore$ Ratio of surface area = 1 : 9.

Example 60 :

A rectangular piece of paper is 71 cm long and 10 cm wide. A cylinder is formed by rolling the paper along its breadth. Find the volume of the cylinder.

Solution :

As the cylinder is made by rolling the paper along its breadth.



$\therefore$ Circumference of the base of cylinder = Width of paper

$\Rightarrow$ $2\pi r = 10$ (r = radius of base of cylinder)

$\Rightarrow$ $r = \frac{5}{\pi}$ cm

Now length of paper = height of cylinder = 71 cm

$$\therefore \quad \text{Volume of cylinder (V)} = \pi r^2 l = \pi \times \frac{25}{\pi \times \pi} \times 71 = \frac{25 \times 7 \times 21}{22}$$

Volume of cylinder (V) = 564.78 cubic cm.

Example 61 :

Two circular cylinders of equal volume have their heights in the ratio 9 : 16. Find the ratio of their radii.

Solution :

Let the radius and height of two cylinder are r_1, h_1 and r_2, h_2 respectively. Let v_1 and v_2 are their volumes.

Now, $v_1 = \pi r_1^2 h_1$ and $v_2 = \pi r_2^2 h_2$

It is given that $v_1 = v_2$

$\therefore \quad \pi r_1^2 h_1 = \pi r_2^2 h_2$

$$\frac{r_1^2}{r_2^2} = \frac{h_2}{h_1} \quad \Rightarrow \quad \frac{r_1}{r_2} = \sqrt{\frac{h_2}{h_1}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$$

$\therefore \quad r_1 : r_2 = 4 : 3$

Example 62 :

Find the volume and surface area of sphere of radius 6.3 cm.

Solution :

Radius of sphere = 6.3 cm

Volume of sphere (V) = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times 6.3 \times 6.3 \times 6.3 = 1047.82\ cm^3$

Surface area of sphere (S) = $4\pi r^2 = 4 \times \frac{22}{7} \times 6.3 \times 6.3 = 498.96\ cm^2$

Example 63 :

Find the volume of a sphere whose surface area is 2464 cm^2.

Solution :

Let radius of sphere = r cm

$\therefore \quad$ Surface area (S) = $4\pi r^2$

$4\pi r^2 = 2464$

$$\Rightarrow \quad r^2 = \frac{2464}{4 \times 22} \times 7 \quad \Rightarrow \quad r = 14\ cm$$

Volume of sphere (V) = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (14)^3 = 11498.67\ cm^3$

Example 64 :

Find the number of lead balls of radius 1 cm each that can be made from a sphere of radius 4 cm.

Solution :

Radius of small lead ball r_s = 1cm.

Radius of big sphere r_b = 4 cm

Let 'n' be the required number of lead balls

$\therefore \quad$ n × volume of 1 lead ball = volume of sphere

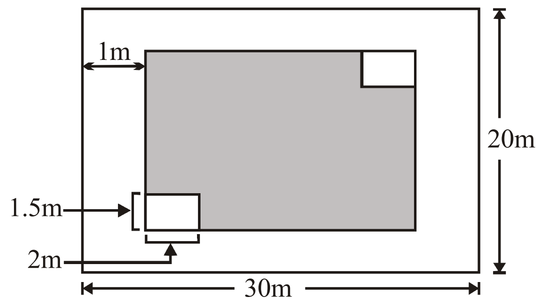
$$\therefore \quad n = \frac{\text{volume of sphere}}{\text{volume of 1 lead ball}} \quad \Rightarrow \quad n = \frac{\frac{4}{3}\pi r_b^3}{\frac{4}{3}\pi r_s^3} = \left(\frac{r_b}{r_s}\right)^3 \Rightarrow n = \left(\frac{4}{1}\right)^3$$

$\therefore \quad n = 64$

$\therefore \quad$ no. of lead balls = 64.

CONCEPT APPLICATION LEVEL - I [NCERT Questions]

Q.1 This is a figure of a rectangular park whose length is 30 m and width is 20 m.



(i) What is the total length of the fence surrounding it ?

(ii) How much land is occupied by the park ?

(iii) There is a path of one metre width running inside alone the perimeter of the park that has to be cemented. If 1 bag of cement is required to cement 4 m^2 area, how many bags of cement would be required to construct the cemented path ?

(iv) There are two rectangular flower beds of size 1.5 × 2m each in the park as shown in the diagram (figure) and the rest has grass on it. Find the area covered by grass.

Sol.

(i) Total length of the fence surrounding it = Perimeter of the park

= 30 m + 20 m + 30 m + 20 m = 100 m

(ii) Land occupied by the park = Area of the park

= 30 × 20 square metre(m^2) = 600 sqaure meters (m^2)

(iii) Area of cemented path = Area of partk – Area of park left after cementing the path.

Now, since path is 1 m wide, so, the rectangular area left after cementing the path

= $\{(30-2) \times (20-2)\}m^2 = (28 \times 18)m^2 = 504\ m^2$

$\therefore$ Area of cemented path = $600\ m^2 - 504\ m^2 = 96\ m^2$

Number of cement bags used = $\dfrac{\text{area of the path}}{\text{area cemented by 1bag}}$

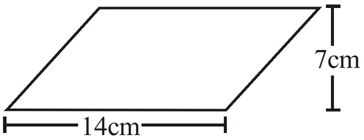
If 1 bag of cement if required to cement 4 m^2 area, then the number of cement bags used = $\dfrac{96}{4} = 24$

(iv) Area of rectangular beds = $2 \times (1.5 \times 2)m^2 = 6\ m^2$

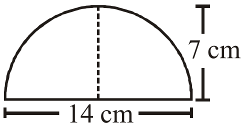
Area of the park left after cementing the path = $504\ m^2$

$\therefore$ Area covered by the grass = $504\ m^2 - 6\ m^2 = 498\ m^2$

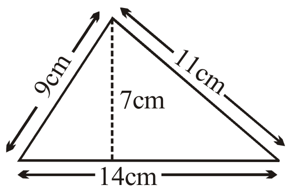
Q.2 (a) Match the following figures with their respective areas in the box.

(i) 

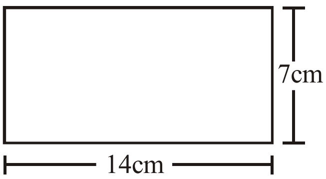
(a) 49 cm^2

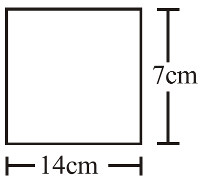
(ii) 

(b) 77 cm^2

(iii) 

(c) 98 cm^2

(iv) 

(v) 

(b) Write the perimeter of each shape.

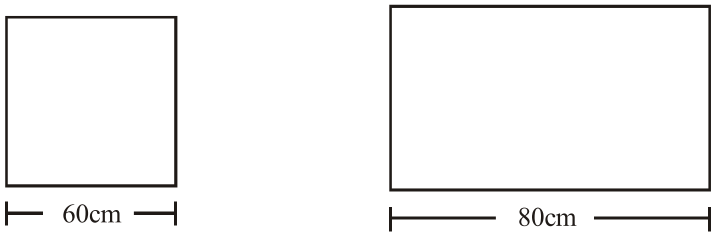
Sol. (a) (i)-c ; (ii)-b ; (iii)-a ; (iv)-c ; (v)-a

(b)

	Shape	Perimeter
(i)	$2 \times (14 + 7) = 42$ cm	
(ii)	$\frac{22}{7} \times 7$ cm + 14 cm = 36 cm	
(iii)	$(14 + 11 + 9)$ cm = 34 cm	
(iv)	$2 \times (14 + 7) = 42$ cm	
(v)	(4×7) cm = 28 cm	

EXERCISE -1

Q.1 A square and a rectangular field with measurements as given in the figure have the same perimeter. Which field has a larger area ?



Sol. Area of the square field = 60 m × 60 m = 3600 m^2

Perimeter of the square field = 4 × 60m = 240 m

$\therefore$ Perimeter of rectangular field = 240 m

$\Rightarrow$ 2(80 + x) = 240

where x m is the breadth of the rectangular field

$\Rightarrow$ $80 + x = \frac{240}{2}$

$\Rightarrow$ 80 + x = 120

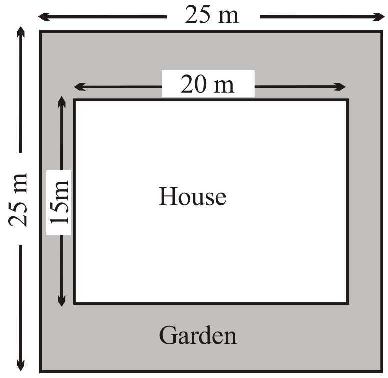
$\Rightarrow$ x = 120 – 80 = 40

$\therefore$ Breadth = 40 m

$\therefore$ Area of rectangular field = l × b = 80 m × 40 m = 3200 m^2

So, the square field has a larger area.

Q.2 Mrs. Kaushik has a square plot with the measurement as shown in the figure. She wants to construct a house in the middle of the plot. A garden is developed around the house. Find the total cost of developing a garden around the house at the rate of ₹ 55 per m^2.



Sol. Area of the square plot = 25 × 25 m^2 = 625 m^2

Area of the house = 20 × 15 m^2 = 300 m^2

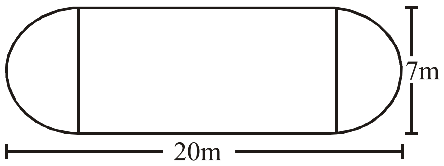
$\therefore$ Area of the garden = Area of the square plot – Area of the house

= 625 m^2 – 300 m^2 = 325 m^2

$\because$ The cost of developing the garden per square metre = ₹ 55.

$\therefore$ Total cost of developing the garden = ₹ 325 × 55 = ₹ 17,875

Q.3 The shape of a garden is rectangular in the middle and semicircular at the ends as shown in the diagram. Find the area and the perimeter of this garden. (Length of rectangle is 20 – (3.5 + 3.5) metres).



Sol. Area of the garden = Area of the rectangle + Area of the two semicircular ends

$$= \{20 - (3.5 + 3.5)\} \times 7\text{m}^2 + 2 \times \frac{1}{2}\pi\left(\frac{7}{2}\right)^2 \text{m}^2$$

$$= 13 \times 7 \text{ m}^2 + \frac{49}{4}\pi \text{ m}^2 = 91 \text{ m}^2 + \frac{49}{4} \times \frac{22}{7} \text{m}^2 = 91 \text{ m}^2 + \frac{77}{2}\text{m}^2 = \frac{259}{2}\text{m}^2 = 129.5 \text{ m}^2$$

Perimeter of the garden

$$= 2 \times \{20 - (3.5 + 3.5)\} \text{ m} + 2 \times \pi\left(\frac{7}{2}\right)\text{m} = 26 \text{ m} + 2 \times \frac{22}{7} \times \frac{7}{2}\text{m} = 26 \text{ m} + 22 \text{ m} = 48 \text{ m}$$

Q.4 A flooring tile has the shape of a parallelogram whose base is 24 cm and the corresponding height is 10 cm. How many such tiles are required to cover a floor of area 1080 m^2 ? (If required you can split the tiles in whatever way you want to fill up the corners).

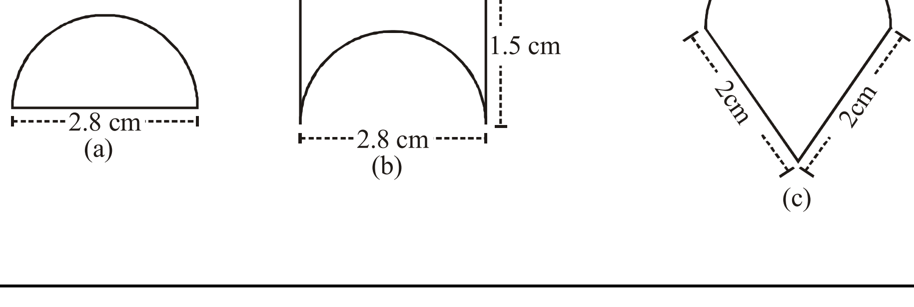
Sol. Area of a flooring tile = 24×10 cm^2 = 240 cm^2

Area of the floor = 1080 m^2 = $1080 \times 100 \times 100$ cm^2

$\because$ 1m^2 = 100×100 cm^2

$\therefore$ Number of tiles required = $\dfrac{\text{Area of the floor}}{\text{Area of a flooring file}} = \dfrac{1080 \times 100 \times 100}{240} = 45000$

Q.5 An ant is moving around a few food pieces of different shapes scattered on the floor. For which food-piece would the ant have to take a longer round ? Remember, circumference of a circle can be obtained by using the expression c = 2πr, where r is the radius of the circle.



Sol. (a) Perimeter of the food piece $= \pi\left(\frac{2.8}{2}\right)$ cm $+ 2.8$ cm

$$= \frac{22}{7} \times 1.4 \text{ cm} + 2.8 \text{ cm} = 4.4 \text{ cm} + 2.8 \text{ cm} = 7.2 \text{ cm}$$

(b) Perimeter of the food piece $= 2.8$ cm $+ 1.5$ cm $+ 1.5$ cm $+ \pi\left(\frac{2.8}{2}\right)$ cm

$$= 5.8 \text{ cm} + \frac{22}{7} \times 1.4 \text{ cm} = 5.8 \text{ cm} + 4.4 \text{ cm} = 10.2 \text{ cm}$$

(c) Perimeter of the food piece $= 2$ cm $+ 2$ cm $+ \pi\left(\frac{2.8}{2}\right)$ cm

$$= 4 \text{ cm} + \frac{22}{7} \times 1.4 \text{ cm} = 4 \text{ cm} + 4.4 \text{ cm} = 8.4 \text{ cm}$$

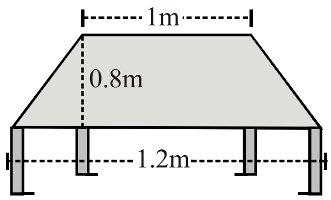
Therefore, the ant would have to take a longer round for food piece (b).

EXERCISE -2

Q.1 The shape of the top surface of a table is a trapezium. Find its area, if its parallel sides are 1 m and 1.2 m and perpendicular distance between them is 0.8 m.

Sol. Area of the top surface of the table

$$= \frac{1}{2}h(a+b) = \frac{1}{2} \times 0.8 \times (1.2 + 1) = 0.88 \text{ m}^2$$



Q.2 The area of a trapezium is 34 cm^2 and the length of one of the parallel sides is 10 cm and its height is 4 cm. Find the length of the another parallel side.

Sol. Area of trapezium $= \frac{1}{2}h(a+b)$

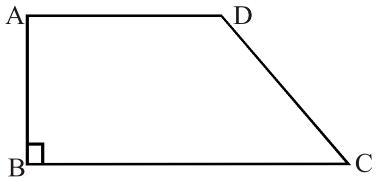
$\Rightarrow \quad 34 = \frac{1}{2} \times 4(10 + b) \quad \Rightarrow \quad 34 = 2 \times (10 + b)$

$\Rightarrow \quad 10 + b = \frac{34}{2} \quad \Rightarrow \quad 10 + b = 17$

$\Rightarrow \quad b = 17 - 10 \quad \Rightarrow \quad b = 7$ cm

Hence, the length of another parallel side is 7 cm

Q.3 Length of the fence of a trapezium shaped field ABCD is 120 m. If BC = 48 m, CD = 17 m and AD = 40 m, find the area of this field. Side AB is perpendicular to the parallel sides AD and BC.



Sol. Fence of the trapezium shaped field.

ABCD = 120 m

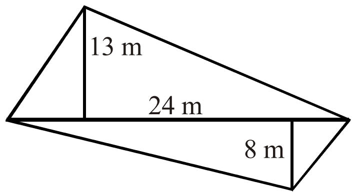
$\Rightarrow$ AB + BC + CD + DA = 120 $\Rightarrow$ AB + 48 + 17 + 40 = 120

$\Rightarrow$ AB + 105 = 120 $\Rightarrow$ AB = 120 = 105

$\Rightarrow$ AB = 15 m

$\therefore$ Area of the field $= \dfrac{(BC + AD) \times AB}{2} = \dfrac{(48 + 40) \times 15}{2} = 660\ m^2$

Q.4 The diagonal of a quadrilateral shaped field is 24 m and the perpendiculars dropped on it from the remaining opposite vertices are 8 m and 13 m. Find the area of the field.

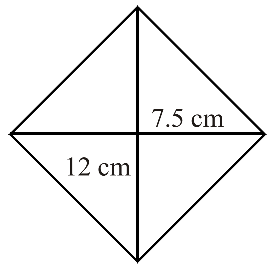


Sol. Area of the field $= \dfrac{24 \times (8 + 13)}{2} = \dfrac{24 \times 21}{2}$

$= 12 \times 21 = 252\ m^2$

Q.5 The diagonals of a rhombus are 7.5 cm and 12 cm. Find its area.

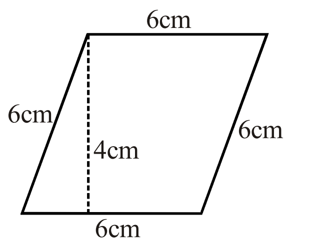
Sol. Area of the rhombus



$= \dfrac{1}{2} \times d_1 \times d_2 = \dfrac{1}{2} \times 7.5 \times 12 = 45\ cm^2$

Q.6 Find the area of a rhombus whose side is 6 cm and whose altitude is 4 cm. If one of its diagonals is 8 cm long, find the length of the other diagonal.

Sol. Area of the rhombus = base × altitude = $6 \times 4 = 24\ cm^2$



Area of the rhombus $= \dfrac{1}{2} \times d_1 \times d_2$

$\Rightarrow 24 = \dfrac{1}{2} \times 8 \times d_2 \quad \Rightarrow 24 = 4d_2 \quad \Rightarrow d_2 = \dfrac{24}{4} = 6$ cm

Hence, the length of the other diagonal is 6 cm.

Q.7 The floor of a building consists of 3000 tiles which are rhombus shaped and each of its diagonals are 45 cm and 30 cm in length. Find the total cost of polishing the floor, if the cost per m^2 is ₹4.

Sol. Area of tile = $\frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 45 \times 30 = 675\ cm^2$

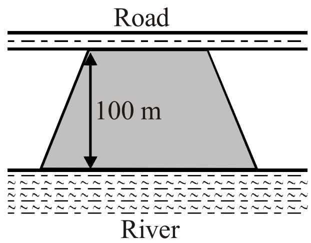
$\therefore$ Area of the floor = $675 \times 3000\ cm^2 = 20{,}25{,}000\ cm^2 = \frac{2025000}{100 \times 100} m^2 = 202.50\ m^2$

The cost of polishing per m^2 = ₹4

$\therefore$ Total cost of polishing the floor = 202.50×4 = ₹ 810.

Q.8 Mohan wants to buy a trapezium shaped field. Its side along the river is parallel to and twice the side along the road. If the area of this field is 10,500 m^2 and the perpendicular distance between the two parallel sides is 100 m, find the length of the side along the river.

Sol. Let the length of the side along the river be 2x m. Then, the length of the side along the road is x m.
Area of the field = 10,500 square metres



$\Rightarrow \frac{(2x + x) \times 100}{2} = 10{,}500 \quad \Rightarrow \quad 150x = 10{,}500$

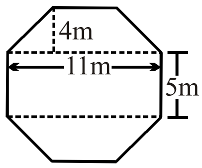
$\Rightarrow x = \frac{10500}{150} \quad \Rightarrow \quad x = 70$

$\Rightarrow 2x = 2 \times 70 = 140$ m

Hence, the length of the side along the river is 140 m.

Q.9 Top surface of a raised platform is in the shape of a regular octagon as shown in the figure. Find the area of the octagonal surface.

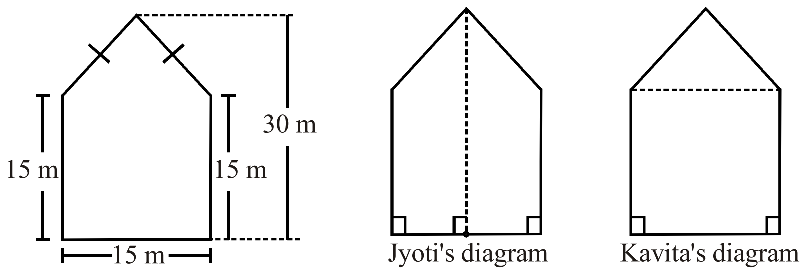
Sol. Area of the octagonal surface
= Area of rectangular portion + 2(Area of trapezoidal portion)



$= 11 \times 5 + 2 \times \left[\frac{(5+11) \times 4}{2}\right] m^2$

$= 55 + 64\ m^2 = 119\ m^2$

Q.10 There is a pentagonal shaped park as shown in the figure. For finding its area Jyoti and Kavita divided it in two different ways.

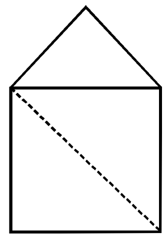


Find the area of this park using both ways. Can you suggest come other way of finding its area?

Sol. **Jyoti's diagram**

$$\text{Area of the park} = 2 \times \left[\frac{(15+30)}{2} \times \frac{15}{2}\right] = \frac{675}{2} = 337.5 \text{ m}^2$$

Kavita's diagram



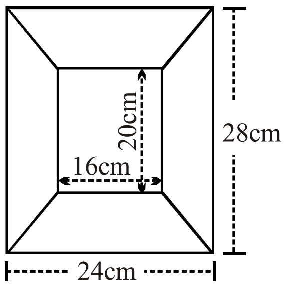
$$\text{Area of the park} = 15 \times 15 \text{ m}^2 + \frac{15 \times (30-15)}{2} \text{ m}^2$$

$$= 225 \text{ m}^2 + \frac{225}{2} \text{ m}^2 = 225 \text{ m}^2 + 112.5 \text{m}^2 = 337.5 \text{ m}^2$$

Another way of finding the area

$$\text{Area of the park} = \frac{15 \times (30-15)}{2} \text{m}^2 + 2 \times \left[\frac{15 \times 15}{2}\right] \text{m}^2 = 337.5 \text{ m}^2$$

Q.11 Diagram of the adjacent picture frame has outer dimensions = 24 cm × 28 cm and inner dimension 16 cm × 20 cm. Find the area of each section of the frame, if the width of each section is same.



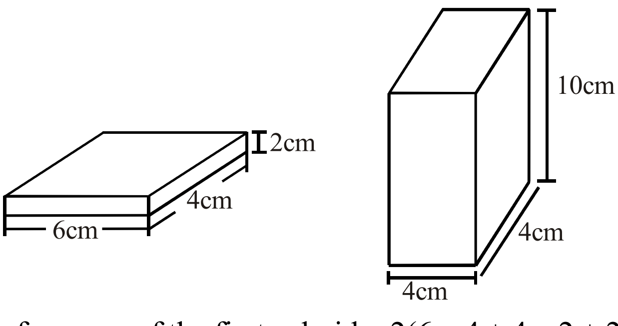
Sol. Area of the right section of the frame $= \dfrac{(28+30) \times \frac{1}{2}(24-16)}{2} \text{cm}^2 = \dfrac{48 \times 4}{2} \text{cm}^2 = 96 \text{ cm}^2$

Similarly, area of the left section of the frame = 96 cm^2

Area of the upper section of the frame $= \dfrac{(24+16) \times \frac{1}{2}(28-20)}{2} \text{cm}^2 = \dfrac{40 \times 4}{2} \text{cm}^2 = 80 \text{ cm}^2$

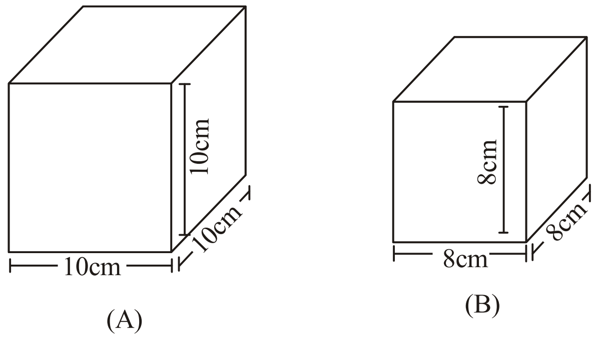
Similarly, area of the lower section of the frame = 80 cm^2

Q.12 Find the total surface area of the following cuboids (figure) :



Sol. Total surface area of the first cuboid = 2(6 × 4 + 4 × 2 + 2 × 6) = 2(24 + 8 +12) = 88 cm^2
Total surface area of the second cuboid = 2(4 × 4 + 4 × 10 + 10 × 4) = 2(16 + 40 + 40) = 192 cm^2

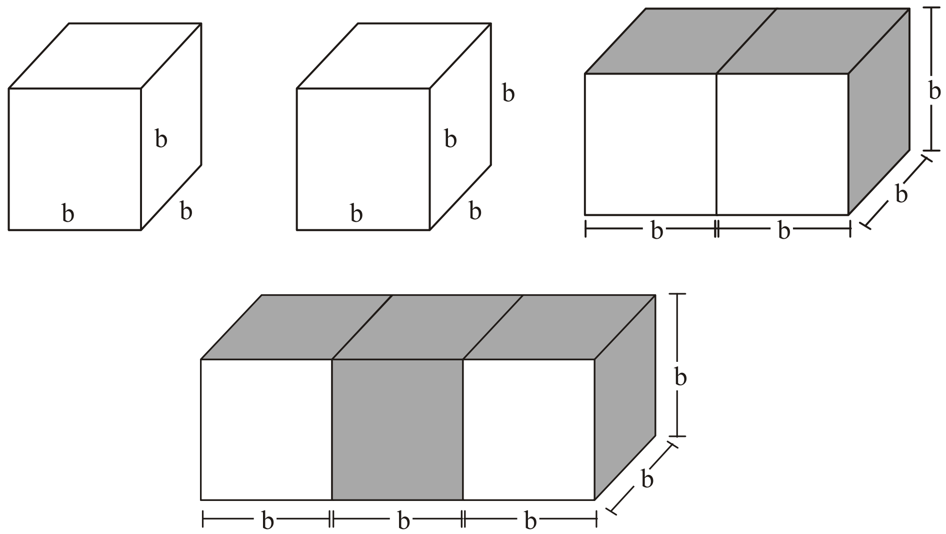
Q.13 Find the surface area of cube A and lateral surface area of cube B.



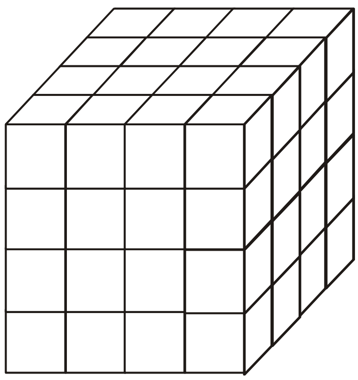
Sol. Surface area of cube (A) = $6\ell^2 = 6(10)^2\ cm^2 = 600\ cm^2$

Lateral surface area of cube (B) = $2 \times (8 + 8) \times 8\ cm^2 = 256\ cm^2$

Q.14 (i) Two cube each with side b are joined to form a cuboid (figure). What is the surface area of this cuboid is it $12b^2$? Is the surface area of cuboid formed by joining three such cubes, $18b^2$? Why?



(ii) How will you arrange 12 cubes of equal length to form a cuboid of smallest surface area?



(iii) After the surface area of a cube is painted, the cube is cut into 64 smaller cubes of same dimensions (figure).

How many have no face painted? 1 face painted? 2 faces painted? 3 faces painted?

Sol. (i) By joining two cubes each with side b, we get a cuboid where,

length (L) = b + b = 2b units

breadth (B) = b units

and height (H) = b units

Surface are of this cuboid

$= 2\,[2b \times b + b \times b + b \times 2b]$ square units

$= 2[2b^2 + b^2 + 2b^2]$ square units

$= 10\,b^2$ square units

$\neq 12b^2$. (No, it is not equal to $12b^2$)

Again, by joining three cubes each with side b, we get a cuboid with

length (L) = b + b + b = 3b units

breadth (B) = b units

and height (H) = b units

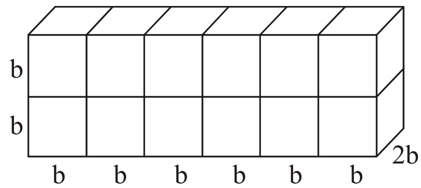
$\therefore$ Surface area of the cuboid formed by joining the three cubes

$= 2 \times [3b \times b + b \times b + b \times 3b]$ square units

$= 2 \times [3b^2 + b^2 + 3b^2]$ square units

$= 14b^2$ square units

$\neq 18b^2$. (No, it is not equal to $18b^2$)



(ii) In first arrangement,

L = 6b ; B = b ; H = 2b

$\therefore$ Surface area

$= 2 \times (6b \times b + b \times 2b + 2b \times 6b) = 2 \times (6b^2 + 2b^2 + 12\,b^2) = 40\,b^2$

In second arrangement,

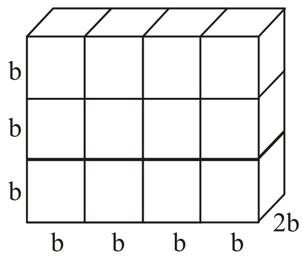
L = 4b ; B = b ; H = 3b

$\therefore$ Surface area

$= 2 \times (4b \times b + b \times 3b + 3b \times 4b)$

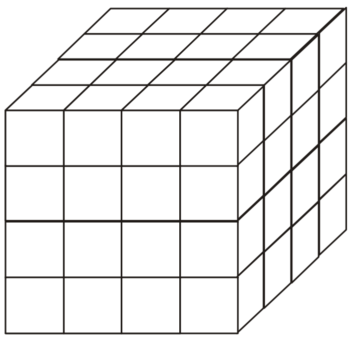
$= 2 \times (4b^2 + 3b^2 + 12b^2)$

$= 38\,b^2$



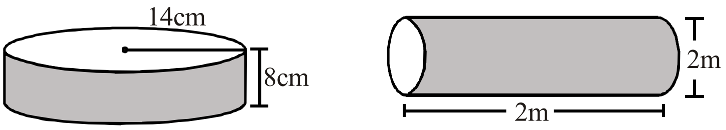
Hence, for smallest surface area, the latter arrangement must be made.

(iii) 16 cubes have no faces painted. 24 cubes have 1 face painted. 16 cubes have 2 faces painted.



8 cubes have 3 faces painted

Q.15 Find total surface area of the following cylinders (figure).



Sol. For first cylinder, r = 14 cm, h = 8 cm

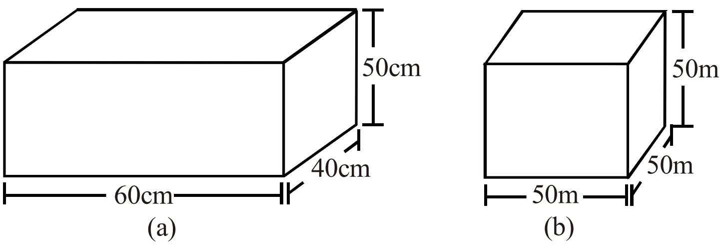
$\therefore$ Total surface area of the cylinder $= 2\pi r\,(r + h) = 2 \times \frac{22}{7} \times 14 \times (14 + 8)\ cm^2 = 1936\ cm^2$

For second Cylinder $r = \frac{2}{2}\ m = 1\ m$, h = 2m

$\therefore$ Total surface area of the cylinder $= 2\pi r\,(r + h) = 2 \times \frac{22}{7} \times 1 \times (1 + 2)\ m^2$

$$= \frac{132}{7}\ m^2 = 18\frac{6}{7}\ m^2$$

Q.16 There are two cuboidal boxes as shown in adjoining figure. Which box requires the lesser amount of material to make ?



Sol. First Cuboidal Box

l = 60 cm ; b = 40 cm ; h = 50 cm

$\therefore$ Total surface area $= 2(lb + bh + hl) = 2\,(60 \times 40 + 40 \times 50 + 50 \times 60)\ cm^2$

$= 2(2400 + 2000 + 3000)\ cm^2 = 2(7400)\ cm^2 = 14800\ cm^2$

Second Cuboidal Box, l = 50 cm ; b = 50 cm ; h = 50 cm

$\therefore$ Total surface area $= 2(lb + bh + hl) = 2(50 \times 50 + 50 \times 50 + 50 \times 50)\ cm^2$

$= 2(2500 + 2500 + 2500)\ cm^2 = 2(7500)\ cm^2 = 15000\ cm^2$

Hence, the box (a) requires the least amount of material to make.

Q.17 A suitcase with measures 80 cm × 48 cm × 24 cm is to be covered with a tarpaulin cloth. How many metres of trapaulin of width 96 cm is required to cover 100 such suitcases ?

Sol. Total surface area of the suitcases

$= 2(80 \times 48 + 18 \times 24 + 24 \times 80)\ cm^2 = 2(3840 + 1152 + 1920) cm^2$

$= 2(6912)\ cm^2 = 13824\ cm^2$

$\therefore$ Length of trapaulin required to cover 1 suitcase

$$= \frac{\text{Total surface area of the suitcase}}{\text{width of trapaulin}} = \frac{13824}{96} = 144\ cm$$

$\therefore$ Length of trapaulin required to cover 100 such suitcases = 144 × 100 cm = 14400 cm = 144 m

Hence, 144 m of trapaulin is required.

Q.18 Find the side of a cube whose surface area is 600 cm^2.

Sol. Let the side of the cube be a cm.

Then, Total surface area of the cube

$= 6a^2$

According to the question,

$6a^2 = 600$

$\Rightarrow \quad a^2 = \frac{600}{6}$

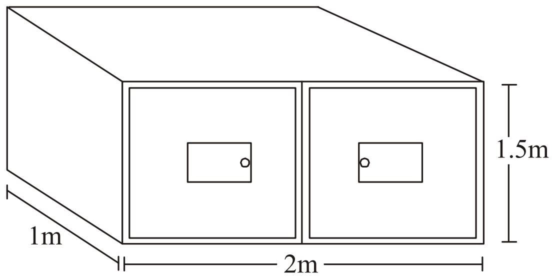
$\Rightarrow \quad a^2 = 100$

$\Rightarrow \quad a = \sqrt{100}$

$\Rightarrow \quad a = 10$ cm

Hence, the side of the cube is 10 cm.

Q.19 Rukhsar painted the outside of the cabinet of measure 1m × 2m × 1.5 m. How much surface area did she cover if the painted all except the bottom of the cabinet.



Sol. $\ell = 1$m; $b = 2$m; $h = 1.5$ m

Required area $= 2(\ell \times b + b \times h + h \times \ell) - \ell \times b$

$= 2(1 \times 2 + 2 \times 1.5 + 1.5 \times 1)\ m^2 - (1 \times 2)\ m^2$

$= 13\ m^2 - 2m^2 = 11\ m^2$

Hence, she covered $11m^2$ of surface area.

Q.20 Daniel is painting the walls and ceiling of a cubodial hall with length, breadth and height of 15m, 10m and 7m respectively. From each can of paint 100 m^2 of area is painted. How many cans of point will she need to paint the room?

Sol. $\ell = 15$ m

$b = 10$ m

$h = 7$ m

Surface area to be painted

$= 2(\ell \times b + b \times h + h \times \ell) - \ell \times b$

$= 2(15 \times 10 + 10 \times 7 + 7 \times 15)\ m^2 - (15 \times 10)\ m^2$

$= 2(150 + 70 + 150)\ m^2 - 150\ m^2$

$= 2(325)m^2 - 150\ m^2$

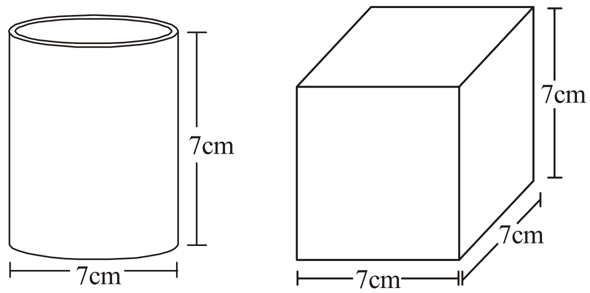
$= 650\ m^2 - 150\ m^2$

$= 500\ m^2$

$\therefore \quad$ Number of cans needed $= \frac{\text{Surface area to be painted}}{\text{Area painted by 1 can}} = \frac{500}{100} = 5$

Hence, she will need 5 cans of paint to paint the room.

Q.21 Describe how the two figures at the right are alike and how they are different. Which box has larger lateral surface area?



Sol. Similarity $\rightarrow$ Both have the same heights.
Difference $\rightarrow$ One is cylinder, the other is a cube;
Cylinder is a solid obtained by revolving a rectangular area about its one side whereas a cube is a solid enclosed by six faces; a cylinder has two circular faces whereas a cube has six square faces.

For first figure

$$r = \frac{7}{2}\text{cm}; \qquad h = 7\text{cm}$$

$\therefore$ Lateral surface area

$$= 2\pi rh = 2 \times \frac{22}{7} \times \frac{7}{2} \times 7 = 154 \text{ cm}^2$$

For second figure

$\ell = 7\text{cm}; \ b = 7 \text{ cm}; \ h = 7\text{cm}$

$\therefore$ Lateral surface area $= 4\ell^2 = 4 \times (7)^2 = 196 \text{ cm}^2$

Hence, the second box has the larger lateral surface area.

Q.22 A closed cylindrical tank of radius 7m and height 3m is made from a sheet of metal. How much sheet of metal is required?

Sol. $r = 7\text{m}; h = 3\text{m}$

$\therefore$ Total surface area $= 2\pi r(r + h)$

$$= 2 \times \frac{22}{7} \times 7 \times (7 + 3)$$

Hence, 440 m^2 of metal sheet is required.

Q.23 The lateral surface area of a hollow cylinder is 4224 cm^2. It is cut along its height and formed a rectangular sheet of width 33 cm. Find the perimeter of rectangular sheet?

Sol. Lateral surface area of the hollow cylinder = 4224 cm^2

$\Rightarrow$ Area of the rectangular sheet = 4224 cm^2

Lenght × 33 = 4224

$$\Rightarrow \text{Length} = \frac{4224}{33}$$

$\Rightarrow$ Length = 128 cm

$\therefore$ Perimeter of the rectangular sheet = 2(Length + Breadth)

$$= 2(128 + 33) \text{ cm} = 2(161) \text{ cm} = 322 \text{ cm}$$

Hence, the perimeter of the rectangular sheet is 322 cm.

Q.24 A road roller takes 750 complete revolutions to move once over to level a road. Find the area of the road if the diameter of a road roller is 84 cm and length is 1m.

Sol. Diameter of the road roller

$= 84$ cm

$\therefore$ Radius (r) of the road roller

$= \frac{84}{2}$ cm $= 42$ cm

Length (h) of the road roller

$= 1$m $= 100$ cm

$\therefore$ Lateral surface area of the road roller $= 2\pi rh$

$= 2 \times \frac{22}{7} \times 42 \times 100$

$= 26{,}400$ cm^2

$\therefore$ Area of the road covered in 1 complete revolution

$= 26{,}400$ cm^2

$\therefore$ Area of the road covered in 750 complete revolutions

$= 26{,}400 \times 750$ cm^2

$= 1{,}98{,}00{,}000$ cm^2

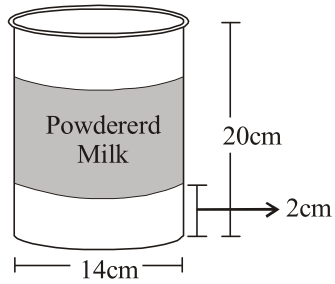
$= \frac{1{,}98{,}00{,}000}{100 \times 100}$ m^2

$= 1{,}980$ m^2.

Q.25 A company package its milk powder in cylindrical container whose base has a diameter of 14 cm and height 20 cm. Company places a label around the surface of the container (as shown in the figure). If the label is placed 2cm from top and bottom, what is the area of the label.

Sol. For a cylindrical container

Diameter of the base $= 14$ cm



$\therefore$ Radius of the base (r) $= \frac{14}{2}$ cm $= 7$ cm

Height (h) $= 20$ cm

$\therefore$ Curved surface area of the container

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times 7 \times 20$

$= 880$ cm^2

$\therefore$ Surface area of the label

$= 880 \text{ cm}^2 - 2\left(2 \times \frac{22}{7} \times 7 \times 2\right) \text{ cm}^2$

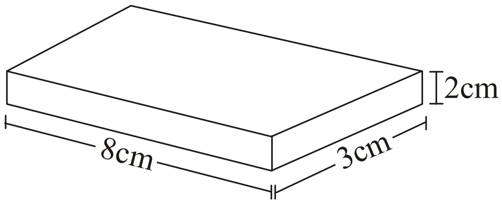
$= 880$ cm$^2 - 176$ cm^2

$= 704$ cm^2

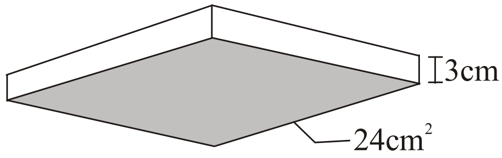
Hence, the surface area of the label is 704 cm^2.

Q.26 Find the volume of the following cuboids (figure).

(i)



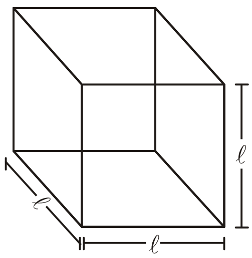
(ii)



Sol. (i) Volume of the cuboid

$= (8 \times 3 \times 2)\ cm^3$

$= 48\ cm^3$



(ii) Volume of the cuboid

$= (24 \times 3)\ cm^3$

$= 72\ cm^3$

Q.27 Find the volume of the following cubes:
(a) with a side 4 cm
(b) with a side 1.5 m

Sol. (a) Volume of the cube $= 4 \times 4 \times 4\ cm^3 = 64\ cm^3$

(b) Volume of the cube $= 1.5 \times 1.5 \times 1.5\ m^3 = 3.375\ m^3$

Q.28 Arrange 64 cube of equal size in as many ways as you can to form a cuboid. Find the surface area of each arrangement. Can solid shapes of same volume have same surface area?

Sol. Some arrangements are as follows:

	In length	In breadth	In height
(i)	64	1	1
(ii)	32	2	1
(iii)	16	2	2
(iv)	16	4	1
(v)	8	4	2
(vi)	4	4	4, etc.

Surface area in arrangement

(i) $= 2 \times (64 \times 1 + 1 \times 1 + 1 \times 64) = 258$ square units

(ii) $= 2 \times (32 \times 2 + 2 \times 1 + 1 \times 32) = 196$ square units

(iii) $= 2 \times (16 \times 2 + 2 \times 2 + 2 \times 16) = 136$ square units

(iv) $= 2 \times (16 \times 4 + 4 \times 1 + 1 \times 16) = 168$ square units

(v) $= 2 \times (8 \times 4 + 4 \times 2 + 2 \times 8) = 112$ square units

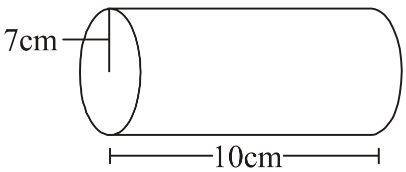
(vi) $= 2 \times (4 \times 4 + 4 \times 4 + 4 \times 4) = 96$ square units

Also, volume of the cuboid obtained in each case in 64 cubic units.

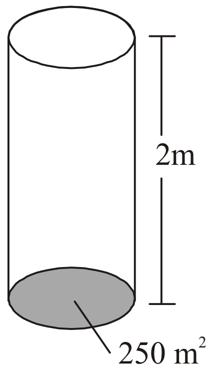
So, No! we cannot say that solid shape of same volume need to have same surface area.

Q.29 Find the volume of the following cylinders:

(i)



(ii)



Sol. $r = 7$ cm, $h = 10$ cm

(i) Volume of the cylinder

$= \pi r^2 h$

$= \pi \times 7^2 \times 10$

$= \frac{22}{7} \times 7 \times 7 \times 10$

$= 1540 \text{ cm}^3$

(ii) Volume of the cylinder

= Area of base × height

$= (250 \times 2) \text{ m}^3$

$= 500 \text{ m}^3$.

Q.30 Given a cylindrical tank, in which situation will you find surface area and in which situations volume.

(a) To find how much it can hold.

(b) Number of cement bags required to plaster it.

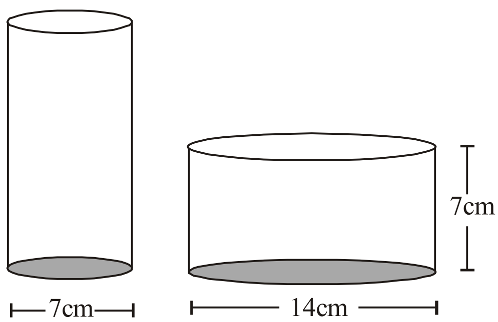
(c) To find the number of smaller tanks that can be filled with water from it.

Sol. (a) Volume

(b) Surface area

(c) Volume.

Q.31 Diameter of cylinder A is 7 cm, and the height is 14 cm. Diameter of cylinder B is 14 cm and height is 7 cm. Without doing any calculations can you suggest whose volume is greater? Verify it by finding the volume of both the cylinders. Check whether the cylinder with greater volume also has greater surface area?



(A) (B)

Sol. Volume of cylinder B is greater.

For Cylinder A

$$r = \frac{7}{2}\,\text{cm}$$

h = 14 cm

$\therefore$ Volume = $\pi r^2 h$

$$\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 14$$

$= 539\ \text{cm}^3$

For Cylinder B

$$r = \frac{14}{2}\,\text{cm} = 7\text{cm}$$

h = 7 cm

$\therefore$ Volume = $\pi r^2 h$

$$= \frac{22}{7} \times 7 \times 7 \times 7$$

$= 1078\ \text{cm}^3$.

By actual calculation of volumes of both, it is verified that the volume of cylinder B is greater.

For Cylinder A

Surface area = $2\pi r\,(r + h)$

$$= 2 \times \frac{22}{7} \times \frac{7}{2} \times \left(\frac{7}{2}+14\right)$$

$$= 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{35}{2}$$

$= 385\ \text{cm}^2$

For Cylinder B

Surface area = $2\pi r\,(r + h)$

$$= 2 \times \frac{22}{7} \times 7 \times (7 + 7)$$

$$= 2 \times \frac{22}{7} \times 7 \times 14$$

$= 616\ \text{cm}^2$

By actual calculation of surface area of both, we observe that the cylinder with greater volume has greater surface area.

Q.32 Find the height of a cuboid whose base area is 180 cm^2 and volume is 900 cm^3?

Sol. Height of the cuboid

$$= \frac{\text{Volume of the cuboid}}{\text{Base area of the cuboid}} = \frac{900}{180}$$

= 5 cm

Q.33 A cuboid is of dimensions 60 cm × 54 cm × 30 cm. How many small cubes with side 6 cm can be placed in the given cuboid?

Sol. Volume of the cuboid

$= 60 \times 54 \times 30 \text{ cm}^3$

$= 97200 \text{ cm}^3$

Volume of a small cube $= 6 \times 6 \times 6 \text{ cm}^3 = 216 \text{ cm}^3$

$\therefore$ Number of small cubes that can be placed in the given cuboid

$$= \frac{\text{Volume of the cuboid}}{\text{Volume of a small cube}} = \frac{97200}{216} = 450$$

Hence, 450 small cubes can be placed in the given cuboid.

Q.34 Find the height of the cylinder whose volume is 1.54 m^3 and diameter of the base is 140 cm?

Sol. $\because$ Diameter of the base = 140 cm

$\therefore$ Radius of the base (r)$= \frac{140}{2}$ cm = 70 cm

$\therefore$ Area of the base $= \pi r^2 = \frac{22}{2} \times 70 \times 70 = 15400 \text{ cm}^2$

Volume of the cylinder $= 1.54 \text{ m}^3$

$= 1.54 \times 100 \times 100 \times 100 \text{ cm}^3$

$= 1540000 \text{ cm}^3$

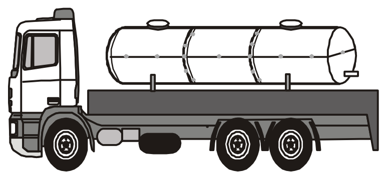
$\therefore$ Height of the cylinder $= \frac{\text{Volume of the cylinder}}{\text{Area of the base of the cylinder}}$

$$= \frac{1540000}{15400} = 100 \text{ cm} = 1 \text{ m}$$

Hence, the height of the cylinder is 1m.

Q.35 A milk tank is in the form of cylinder whose radius is 1.5 m and length is 7 m. Find the quantity of milk in litres that can be stored in the tank?

Sol. For milk tank



$r = 1.5$ m

$h = 7$m

$\therefore$ Capacity $= \pi r^2 h$

$$= \frac{22}{7} \times 1.5 \times 1.5 \times 7$$

$$= \frac{22}{7} \times \frac{15}{10} \times \frac{15}{10} \times 7$$

$= 49.5 \text{ m}^3$

$= 49.5 \times 1000$ L

$[\because 1\text{m}^3 = 1000 \text{ L}]$

$= 49500$ L

Hence, the quantity of milk that can be stored in the tank is 49500 litres.

Q.36 If each edge of a cube is doubled,
(i) How many times will its surface area increase?
(ii) How many times will its volume increase?

Sol. Let the original edge of the cube be a cm.
Then, its new edge = 2a cm

(i) Original surface area of the cube = $6a^2$ cm^2
New surface area of the cube $= 6\,(2a)^2\ cm^2$
$= 24\ a^2\ cm^2 = 4\,(6a^2\ cm^2)$
= 4 original surface area,

Hence, its surface area will increase 4 times.

(ii) Original volume of the cube = a^3 cm^3
New volume of the cube $= (2a)^3\ cm^3 = 8a^3\ cm^3$
= 8 original volume of the cube.

Hence, its volume will increase 8 times.

Q.37 Water is pouring into a cuboidal reservoir at the rate of 60 litres per minute. If the volume of reservoir is 108 m^3, find the number of hours it will take to fill the reservoir.

Sol. Volume of reservoir = 108 m^3 = 108 × 1000 L = 108000 L
Water poured per minute = 60 L

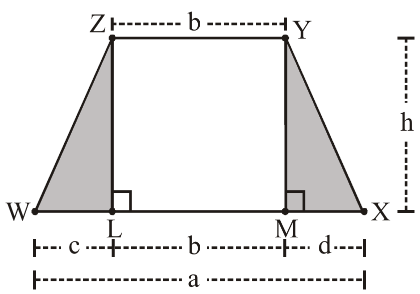
$\therefore$ Time taken to full the reservoir $= \dfrac{\text{Volume of the reservoir}}{\text{Water poured per minute}}$

$$= \frac{108000}{60}\,m = \frac{108000}{60 \times 60}\ \text{hours}$$

Hence, the number of hours it will take to fill the reservoir is 30.

TRY THESE

Q.1 Nazma's sister also has a trapezium shaped plot. Divide it into three parts as shown (figure).
Show that the area of trapezium WXYZ = h $\frac{(a+b)}{2}$.



Sol. Area of trapezium WXYZ = Area of triangle WLZ + Area of rectangle LMYZ + Area of triangle XMY

$$= \frac{1}{2}c \times h + b \times h + \frac{1}{2}d \times h = \frac{1}{2}\ \{(c + b + d) + b\}h = \frac{h}{2}(a + b).$$

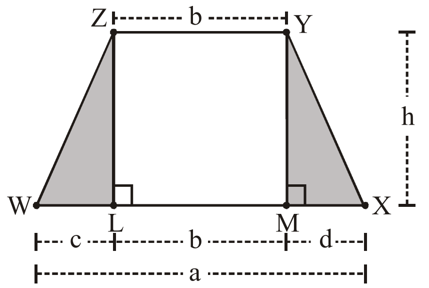
$\because c + b + d = a$

Q.2 If h = 10 cm. c = 6 cm, b = 12 cm, d = 4 cm, find the values of each of its parts separetely and add to find the area WXYZ. Verify it by putting the values of h, a and b in the expression $\frac{h(a+b)}{2}$**.**

Sol. Area of triangle WLZ = $\frac{1}{2} \times c \times h = \frac{1}{2} \times 6 \times 10 = 30\ cm^2$

Area of rectangle LMYZ = $b \times h = 12 \times 10 = 120\ cm^2$

Area of triangle XMY = $\frac{1}{2} d \times h = \frac{1}{2} \times 4 \times 10 = 20\ cm^2$



$\therefore$ Area of trapezium WXYZ

= Area of triangle WLZ + Area of rectangle LMYZ + Area of triangle XMY

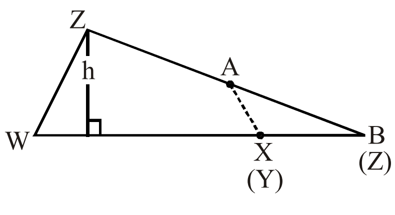
= $30\ cm^2 + 120\ cm^2 + 20\ cm^2 = 170\ cm^2$

Again, area of trapezium WXYZ = $\frac{h}{2}(a+b) = \frac{h}{2}(c+b+d+b)$

$= \frac{h}{2}(c + 2b + d) = \frac{10}{2}(6 + 2 \times 12 + 4) = \frac{10 \times 34}{2} = 170\ m^2$

which is the same as obtained above.

Q.3 What is the length of the base of the larger triangle ? Write an expression for the area of this triangle.

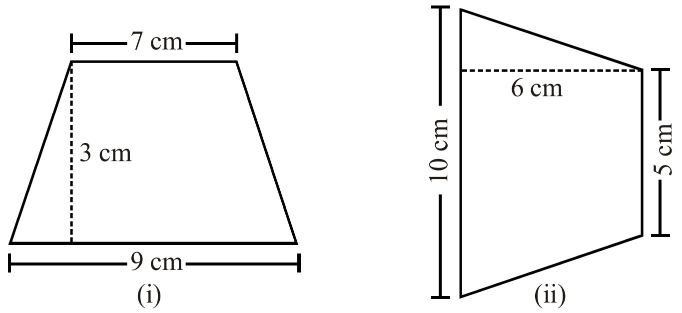


Sol. Length of the base of the larger triangle WBZ = WB = WX + XB

= WX + ZY | $Y \to X, Z \to B$ = a + b

$\therefore$ Area of this triangle = $\frac{\text{Base} \times \text{Height}}{2} = \frac{WB \times h}{2}$

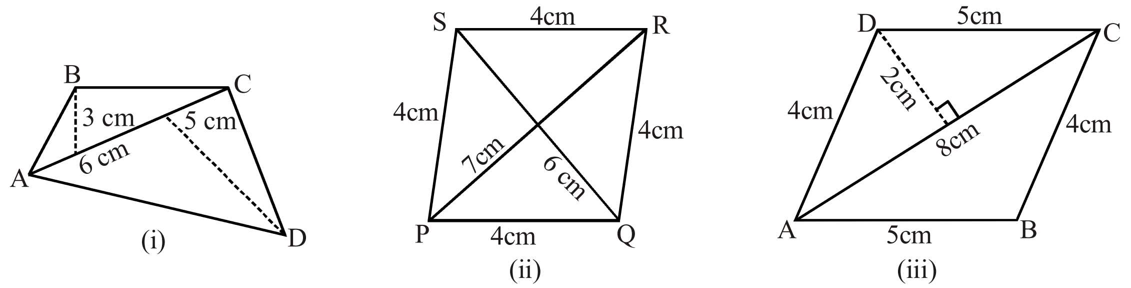
Q.4 Find the area of the following trapeziums (figure given below).



Sol. (i) Area of the trapezium $= \dfrac{(9+7)\times 3}{2} = 24\text{ cm}^2$

(ii) Area of trapezium $= \dfrac{(10+5)\times 6}{2} = 45\text{ cm}^2$

Q.5 Find the area of these quadrilaterals (figures given below).



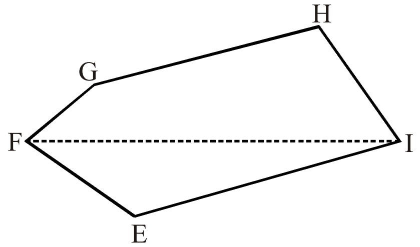
Sol. (i) Area of the quadrilateral ABCD = Area of ΔABC + Area of ΔADC

$$= \frac{1}{2}\times 6\times 3 + \frac{1}{2}\times 6\times 5 = 9 + 15 = 24\text{ cm}^2$$

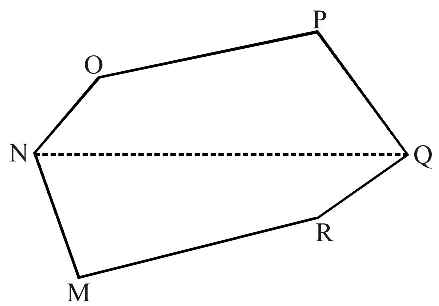
(ii) Area of the quadrilateral PQRS $= \dfrac{1}{2} \times$ diagonal (PR) $\times$ diagonal (QS) $= \dfrac{7\times 6}{2} = 21\text{ cm}^2$

(iii) Area of the quadrilateral ABCD = 2 × area of ΔADC $= 2\times\left(\dfrac{1}{2}\times 8\times 2\right) = 8\times 2 = 16\text{ cm}^2$

Q.6 (i) Divide the following polygons (figures) into parts (triangles and trapezium) to find out its area.

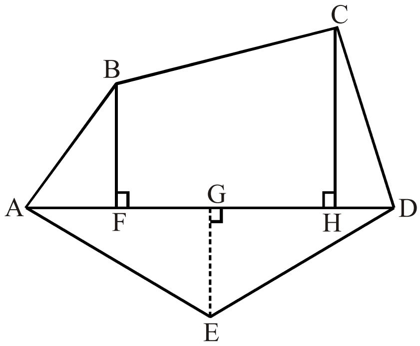


FI is a diagonal of polygon EFGHI



NQ is a diagonal of polygon MNOPQR

(ii) Polygon ABCDE is divided into parts as shown (figure). Find its area if AD = 8 cm, AH = 6 cm, AG = 4 cm, AF = 3 cm and perpendiculars BF = 2 cm, CH = 3 cm, EG = 2.5 cm. Area of polygon ABCDE = area of **ΔAFB + ...**



Area of ΔAFB = $\frac{1}{2}$ × AF × BF = $\frac{1}{2}$ × 3 × 2 = ...

Area of trapezium FBCH = FH × $\frac{(BF+CH)}{2}$ = 3 × $\frac{(2+3)}{2}$ [FH = AH – AF]

Area of ΔCHD = $\frac{1}{2}$ × HD × CH = ...

Area of ΔADE = $\frac{1}{2}$ × AD × DE = ...

So, the area of polygon ABCDE = ...

Sol.(i) Draw GL $\perp$ FI, HM $\perp$ FI and EN $\perp$ FI

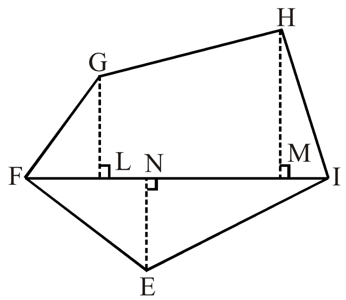
Area of polygon EFGHI

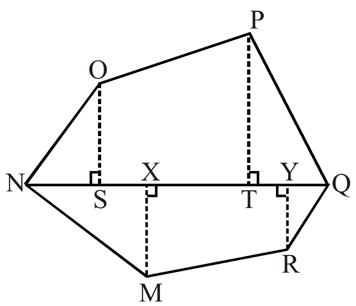
= Area of ΔEFI + Area of ΔFLG + Area of trapezium LMHG + Area of ΔHMI

$= \frac{1}{2} \times FI \times EN + \frac{1}{2} \times FL \times GL + \frac{1}{2} \times (GL + HM) \times LM + \frac{1}{2} \times MI \times MH$

Draw MX $\perp$ NQ, RY $\perp$ NQ, OS $\perp$ NQ and PT $\perp$ NQ

Area of polygon MNOPQR = Area of ΔMXN + Area of trapezium MXYR + Area of ΔRYQ + Area of ΔPTQ + Area of trapezium TPOS + Area of ΔOSN





$= \frac{1}{2} \times MX \times NX + \frac{1}{2} \times (MX + RY) \times XY + \frac{1}{2} \times YQ + YR + \frac{1}{2} \times TQ \times TP + \frac{1}{2} \times (SO + TP) \times ST + \frac{1}{2} \times NS \times OS$

(ii) Area of polygon ABCDE = Area of ΔAFB + Area of trapezium FBCH + Area of ΔCHD + Area of ΔADE

Area of $\Delta AFB = \frac{1}{2} \times AF \times BF = \frac{1}{2} \times 3 \times 2 = 3\ cm^2$

Area of trapezium $FBCH = FH \times \frac{(BF + CH)}{2} = 3 \times \left(\frac{2+3}{2}\right) = 7.5\ cm^2$ $[\because\ FH = AH - AF]$

Areaof $\Delta CHD = \frac{1}{2} \times HD \times CH = \frac{1}{2} \times (AD - AH) \times CH = \frac{1}{2} \times (8-6) \times 3 = 3\ cm^2$

$[\because\ HD = AD - AH]$

Area of $\Delta ADE = \frac{1}{2} \times AD \times GE\ \ \frac{1}{2} \times 8 \times 2.5 = 10\ cm^2$

So, the area of polygon ABCDE = $3\ cm^2 + 7.5\ cm^2 + 3\ cm^2 + 10\ cm^2 = 23.5\ cm^2$